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SENSITIVITY ANALYSIS OF A POLYVINYL CHLORIDE (PVC) MANUFACTURING PLANT

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Abstract:-One has chosen the Polyvinyl Chloride (PVC) industry situated in the Hisar District for the current inquiry. Only when all four units are in good running condition are PVC plants considered to be economically feasible. The system performs at its best when all four of its components are in good working order. It runs at a reduced capacity when three out of four units are functional. The system is in a failed state when two or more units fail. For each of the four units, there are distinct continual failure and repair rates. One repairman is on call around-the-clock. To assess the influence of disappointment/repair rates on MTSF, accessibility, and server during a busy period, precise scenarios must still be generatedwith the assistance ofcharts and tables.

Keywords: MTSF**,** RPGT, Sensitivity Analysis

1. Introduction

PVC is a popular commercial polymer that is used as a primary raw material in a wide range of chemical and petrochemical applications. The four main sub-units in the PVC business are High Impact PVC, Molecular Oriented PVC, Unplasticized PVC, and Chlorinated PVC. In other words, Unplasticized reduces friction while increasing plasticity or viscosity. Rigid PVC is another name for a PVC-U pipe. Unplasticized PVC (PVC-U) has an amorphous structure that is transformed into a layered structure during the production process to create molecularly orientated PVC (PVC-O). Chlorinated PVC is known as C-PVC. Many of the benefits of PVC-U are also found in C-PVC pipes and fittings.PVC-HI, or High Impact PVC, is a plastic. The PVC-HI grade is created by combining PVC-U with an impact modifier that strengthens the pipes' resistance to outside strikes. This sector is divided into a number of components. The framework keeps running for the brief time indicated even if any of its subsystems malfunction. The subsystems are patched prior to entering the bombed condition to avoid the significant separation. A bread plant that was depleting RPGT was the subject of a behavior analysis by Kumar et al. in 2018. Kumar et al. (2019) examined mathematical formulation and behavior analysis in a paper mill washing unit.In their research, Kumar (2018) examined the sensitivity analysis of a 3:4:: remarkable system plant. Kumar (2019)rummage-sale RPGT to perform a sensitivity study on a cold standby architecture composed of two identical units with a server failure and prioritized for preventative maintenance.The current paper is divided into two half, one of which is being used and the other of which is on cold standby. The main distinctions between online and cold standby equipment are the excellent and fully failed modes. Rajbala et al. (2022) looked at the redundancy allocation issue in the cylinder manufacturing factory using a heuristic technique. Rajbala et al. (2019) studied a case study of an EAEP manufacturing facility as part of their work on system modeling and analysis. Kumar et al. (2017) investigated the behavior of the urea nourishment industry. In (2017), Kumar et al. looked at the scientific formulation and profit function of a comestible oil refinery facility. The RPGT technique was used to carry out the mathematical formulation. Different formulas for system parameters are produced by assuming that failure/repair rates are independent and constant. Tables and figures are used to discuss system sensitivity and behavior analysis.

2. Assumptions, Notations & Model Description

- Repair and failures are statistically dependent.
- Repair is perfect &repaired unit is as noble as a novel one.

MTSF: Mean time to system failure

AOS: Availability of the System

SOBP: Server of the busy period

ENIR: Expected Fractional Number of Inspections by repair man

$f_{i/}$ $e_i(2 \leq i \leq 5)$: Constant repair/failure rate of units.

B/$\bar{B}$/b : Unit in working / reduced / failed state.

Figure 1 depicts the service's Transition Diagram, which takes into account the above mentioned assumptions as well as notations.

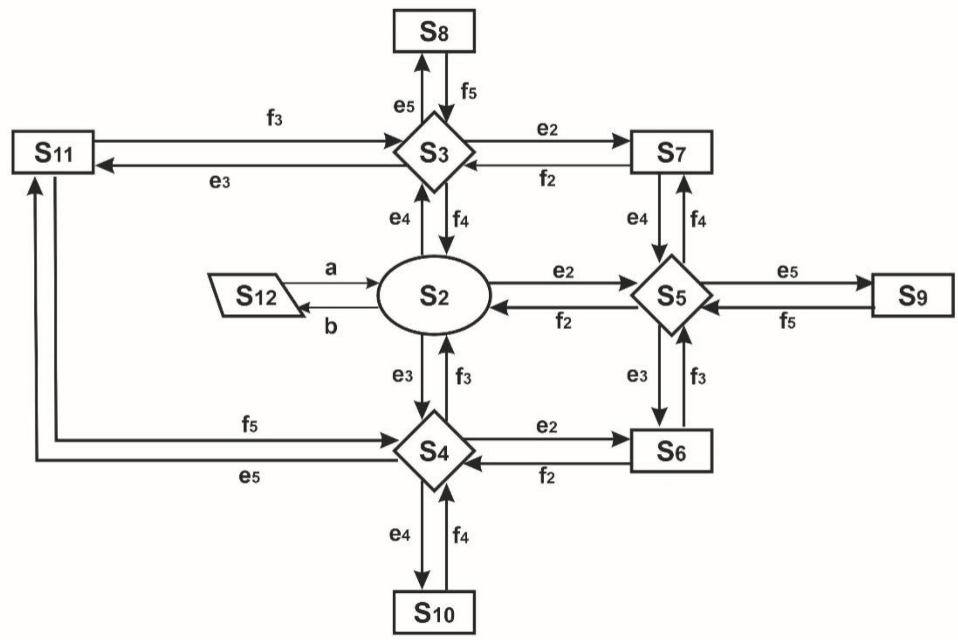

Fig. 1: Transition Diagram of Polyvinyl Chloride Plant

S_2 = ABCD,	S_3 = AB$\bar{C}$D,	S_4 =A$\bar{B}$CD,
S_5 = $\bar{A}$BCD,	S_6= $\overline{AB}$CD,	S_7 = $\bar{A}$B$\bar{C}$D,
S_8 = AB$\overline{CD}$,	S_9 = $\bar{A}$BC$\bar{D}$,	S_{10}= A$\overline{BC}$D,
S_{11}= A$\bar{B}$C$\bar{D}$,	S_{12} = ABCD	

3. Transition Probabilities & Mean Sojourn Times

$q_{i,j}(t)$ **:** p. d. f – “Pdf of the major passage period after a reformative state ‘i’ to a reformative state ‘j’ or to a failed state ‘j’ deprived of waitingsome additional reformative state in (0,t]”.

$p_{i,j}$**:** “Steady state transition probabilities in table 1 after a reformative state ‘i’ to a reformative state ‘j’ deprived of staying any additional reformative state.

Table 1: Transition Probabilities

$q_{i,j}(t)$	$p_{ij} = q^*_{i,j}(t)$
$q_{2,3}(t) = e_4 e^{-(e_2+e_3+e_4+b)t}$ $q_{2,4}(t) = e_3 e^{-(e_2+e_3+e_4+b)t}$ $q_{2,5}(t) = e_2 e^{-(e_2+e_3+e_4+b)t}$ $q_{2,12}(t) = b e^{-(e_2+e_3+e_4+b)t}$	$p_{2,3} = e_4/(e_1+e_3+e_2+b)$ $p_{2,4} = e_3/(e_1+e_3+e_2+b)$ $p_{2,5} = e_2/(e_1+e_2+e_3+b)$ $p_{2,12} = b/(e_3+e_2+e_1+b)$
$q_{3,2}(t) = f_4 e^{-(f_4+e_2+e_5+e_3)t}$ $q_{3,8}(t) = e_5 e^{-(f_4+e_2+e_5+e_3)t}$ $q_{3,7}(t) = e_2 e^{-(f_4+e_2+e_5+e_3)t}$ $q_{3,11}(t) = e_3 e^{-(f_4+e_2+e_5+e_3)t}$	$p_{3,2} = f_4/(f_4 + e_2 + e_5 + e_3)$ $p_{3,8} = e_5/(f_4 + e_2 + e_5 + e_3)$ $p_{3,7} = e_2/(f_4 + e_2 + e_5 + e_3)$ $p_{3,11} = e_3/(f_4 + e_2 + e_5 + e_3)$
$q_{4,2}(t) = f_3 e^{-(f_3+e_2+e_5+e_4)t}$ $q_{4,6}(t) = e_2 e^{-(f_3+e_2+e_5+e_4)t}$ $q_{4,10}(t) = e_4 e^{-(f_3+e_2+e_5+e_4)t}$ $q_{4,11}(t) = e_5 e^{-(f_3+e_2+e_5+e_4)t}$	$p_{4,2} = f_3(f_3 + e_2 + e_5 + e_4)$ $p_{4,6} = e_2/(f_3 + e_2 + e_5 + e_4)$ $p_{4,10} = e_4/(f_3 + e_2 + e_5 + e_4)$ $p_{4,11} = e_5/(f_3 + e_2 + e_5 + e_4)$
$q_{5,2}(t) = f_2 e^{-(f_2+e_3+e_4+e_5)t}$ $q_{5,7}(t) = e_4 e^{-(f_2+e_3+e_4+e_5)t}$ $q_{5,9}(t) = e_5 e^{-(f_2+e_3+e_4+e_5)t}$ $q_{5,6}(t) = e_3 e^{-(f_2+e_3+e_4+e_5)t}$	$p_{5,2} = f_2/(f_2 + e_3 + e_4 + e_5)$ $p_{5,7} = e_4/(f_2 + e_3 + e_4 + e_5)$ $p_{5,9} = e_5/(f_2 + e_3 + e_4 + e_5)$ $p_{5,6} = e_3/(f_2 + e_3 + e_4 + e_5)$
$q_{6,4}(t) = f_2 e^{-(f_3+f_2)t}$ $q_{6,5}(t) = f_3 e^{-(f_3+f_2)t}$	$p_{6,4} = f_2/(f_3 + f_2)$ $p_{6,5} = f_3/(f_3 + f_2)$
$q_{7,3}(t) = f_2 e^{-(f_4+f_2)t}$ $q_{7,5}(t) = f_4 e^{-(f_4+f_2)t}$	$p_{7,3} = f_2/(f_4 + f_2)$ $p_{7,5} = f_4/(f_4 + f_2)$
$q_{8,3}(t) = f_5 e^{-f_5 t}$	$p_{8,3} = 1$
$q_{9,5}(t) = f_5 e^{-f_5 t}$	$p_{9,5} = 1$
$q_{10,4}(t) = f_4 e^{-f_4 t}$	$p_{10,4} = 1$
$q_{11,3}(t) = f_3 e^{-(f_5+f_3)t}$ $q_{11,4}(t) = f_5 e^{-(f_5+f_3)t}$	$p_{11,3} = f_3/(f_5 + f_3)$ $p_{11,4} = f_5/(f_5 + f_3)$
$q_{12,2}(t) = a e^{-at}$	$p_{12,2} = 1$

Table 2: Mean Sojourn Times

$R_i(t)$	$\mu_i=R_i^*(0)$
$R_2(t)= e^{-(e_2+e_3+e_4+b)t}$	$\mu_2 = 1/(e_2 + e_3 + e_4 + b)$
$R_3(t)= e^{-(f_4+e_2+e_5+e_3)t}$	$\mu_3= 1/(f_4 + e_2 + e_5 + e_3)$
$R_4(t)= e^{-(f_3+e_2+e_5+e_4)t}$	$\mu_4= 1/(f_3 + e_2 + e_5 + e_4)$
$R_5(t)= e^{-(f_2+e_3+e_4+e_5)t}$	$\mu_5= 1/(f_2 + e_3 + e_4 + e_5)$
$R_6(t)= e^{-(f_3+f_2)t}$	$\mu_6= 1/(f_3 + f_2)$
$R_7(t)= e^{-(f_4+f_2)t}$	$\mu_7= 1/(f_4 + f_2)$
$R_8(t)= e^{-f_5 t}$	$\mu_8 = 1/f_5$
$R_9(t)= e^{-f_5 t}$	$\mu_9 = 1/f_5$
$R_{10}(t)= e^{-f_4 t}$	$\mu_{10} = 1/f_5$
$R_{11}(t)= e^{-(f_5+f_3)t}$	$\mu_{11} = 1/(f_5 + f_3)$
$R_{12}(t)= e^{-at}$	$\mu_{12} = 1/a$

4. Parametric evaluation

$V_{2,2} = 1$

$V_{2,3}$ = [(2, 3)/(P_1){1-{P_2/ (P_{17})}}{1-{P_3/ (P_{21})}}][1/1-{P_5/(P_4)(P_7)(P_6)(P_9)(P_{11})(P_{10})}]+[(2, 5, 7, 3)/(P_1){1-{P_2/(P_{17})}}{1-{P_3/(P_2)}}(P_{11}){1-{P_9/(P_{18})}}][1/1-{P_4/(P_6)(P_5)(P_9)(P_7) (P_{11})(P_{10})}][1/ 1-{P_{12}/ (1-P_1)(P_5)(P_3)(P_2)(P_7)(P_6)}][1/ 1-{P_{14}/ (P_1)(P_5)(P_5)(P_2)(P_7)(P_9) (P_{11})}][1/ 1-{P_{10}/ (P_{16})}] +[(2,4,6,5,7,3)/(P_1){1-{P_2/(P_{17})}}{1-{P_3/(P_{21})}}{1-{P_6/ (P_{20})}}(P_5)][1/ {1-{P_7/(P_{19})}}{1-{P_9/ (P_{18})}}{1-{P_{10}/s(P_{16}}(P_{11})])}}][1/ 1-{P_4/(P_5) (P_7)(P_6)(P_{10})(P_9)(1-P_{11})}][1/ 1-{P_8/(P_1)(P_2)(P_3)(P_9)(P_{10})(P_{11})}][1/ 1-{P_{12}/ (P_1)(1-P_2) (P_3)(P_6)(P_5)(P_7)}][1/ 1-{P_{13}/(P_1)(P_5)(P_3)(P_2)(P_6)(1-P_{10})(P_{11})}][1/ 1-{P_{14}/ (P_1)(P_3)(P_5) (P_6) (P_7)(P_9)(P_{11})}] + [(2, 4, 11, 3)/(P_1){1-{P_2/ (P_{17})}{1-{P_3/(P_{21})}}(P_5)][1/{1-{P_6/ (P_{20})}}{1-{P_7/ (P_{19})}}][1/ 1-{P_{15}/ (P_1)(P_2)(P_9)(P_{11})(P_5)(P_7)(P_{10})}][1/ 1-{P_8/(P_1)(P_2) (P_9)(P_{10})(P_{10})(P_{11})}] + [(2, 5, 6, 4, 11,3)/(P_1){1-{P_2/(P_{17})}}{1-{P_3/(P_{21})}}(P_5){1- {P_6/(P_{20})}}][1/{1-{P_7/(P_{19})}}{1-{P_9/ (P_{18})}}(P_{11})][1/ 1-{P_4/ (P_9)(P_7)(P_6)(P_5)(P_{10}) (P_{11})}][1/1-{P_8/(P_1)(P_2)(P_3)(P_9)(P_{10})(P_{11})}][1/ 1-{P_{12}/ (P_1)(P_2)(P_5)(P_5)(P_4)(P_7)}] [1/ 1-{P_{13}/(P_1)(P_5)(P_3)(P_2)(P_6)(P_{10})(P_{11})}][1/ 1-{P_{15}/(P_1)(P_2)(P_9)(P_5)(P_7)(P_9)(P_{10})(P_{11})}]

$V_{2,4}$ =…….Continuous

5. Methodology

MTSF (T_0): Reformative un-fizzled states in which framework can transit from early state '2', Earlier any failed state are: 'i' = 2, 4, 3, 5, 12 taking 'ξ' = 2.

$$\text{MTSF } (T_0) = \left[\sum_{i,sr}\left\{\frac{\left\{pr\left(\xi \xrightarrow{sr\,(sff)} i\right)\right\}\mu i}{\Pi_{m_{1\neq\xi}}\{1-V_{\overline{m_1 m_1}}\}}\right\}\right] \div \left[1-\sum_{sr}\left\{\frac{\left\{pr\left(\xi \xrightarrow{sr\,(sff)} \xi\right)\right\}}{\Pi_{m_{2\neq\xi}}\{1-V_{\overline{m_2 m_2}}\}}\right\}\right]$$

$T_0 = (V_{2,2}\,\mu_2 + V_{2,4}\,\mu_4 + V_{2,3}\,\mu_3 + V_{2,5}\,\mu_5 + V_{2,12}\,\mu_{12})/$ [1- {(2, 3, 2) - (2, 4, 2) – (2, 5, 2) - (2, 12, 2)}]= $(V_{2,i}\mu_i)/\ (1- p_{2,3}\,p_{3,2} - p_{2,4}\,p_{4,2} - p_{2,5}\,p_{5,2} - p_{2,12}\,p_{12,2})$; i= 2, 4, 3, 5, 12

AOS (A_0): The reformative states in which outline is reachable are 'j' = 2, 5, 4, 3, 12 and states are 'i' = 2 ≤ i ≤ 12 attractive 'ξ' = '2'

$$A_0 = \left[\sum_{j,sr}\left\{\frac{\{pr\,(\xi^{sr}\rightarrow j)\}fj,\mu j}{\Pi_{m_{1\neq\xi}}\{1-V_{\overline{m_1 m_1}}\}}\right\}\right] \div \left[\sum_{i,s_r}\left\{\frac{\{pr\,(\xi^{sr}\rightarrow i)\}\mu_i^1}{\Pi_{m_{2\neq\xi}}\{1-V_{\overline{m_2 m_2}}\}}\right\}\right]$$

$A_0 = \left[\sum_j V_{\xi,j}\ , f_j\,, \mu_j\right] \div \left[\sum_i V_{\xi,i}\ , f_j\,, \mu_i^1\right]$

$= (V_{2,j}\,\mu_j)/\ D$; (where j = 2, 5, 3, 4, 12)

Where $D = V_{2,i}\mu_i$; (2 ≤ i ≤ 12)

SOBP (B_0): The states in which attendant is hectic are 'j' = 3 ≤ j ≤ 12 after base state ξ = '2'

$$B_0 = \left[\sum_{j,sr}\left\{\frac{\{pr\,(\xi^{sr}\rightarrow j)\},nj}{\Pi_{m_{1\neq\xi}}\{1-V_{\overline{m_1 m_1}}\}}\right\}\right] \div \left[\sum_{i,s_r}\left\{\frac{\{pr\,(\xi^{sr}\rightarrow i)\}\mu_i^1}{\Pi_{m_{2\neq\xi}}\{1-V_{\overline{m_2 m_2}}\}}\right\}\right]$$

$B_0 = \left[\sum_j V_{\xi,j}\ , n_j\right] \div \left[\sum_i V_{\xi,i}\ , \mu_i^1\right]$

$B_0 = (V_{2,j}\,\mu_j)/\ D$; (3 ≤ j ≤ 12)

ENIR (V_0): Reformative states everyplace repairmen do this job 'j' = 3, 4, 5, 12; reformative states are 'i' = 2 ≤ i ≤ 12,

$$V_0 = \left[\sum_{j,sr}\left\{\frac{\{pr\,(\xi^{sr}\rightarrow j)\}}{\Pi_{k_{1\neq\xi}}\{1-V_{\overline{k_1 k_1}}\}}\right\}\right] \div \left[\sum_{i,s_r}\left\{\frac{\{pr\,(\xi^{sr}\rightarrow i)\}\mu_i^1}{\Pi_{k_{2\neq\xi}}\{1-V_{\overline{k_2 k_2}}\}}\right\}\right]$$

$V_0 = \left[\sum_j V_{\xi,j}\right] \div \left[\sum_i V_{\xi,i}\ , \mu_i^1\right]$

$= (V_{2,j}\,\mu_j)/\ D$; (Where; j = 3, 4, 5, 12)

6. **Sensitivity Analysis:** Also, the above subsequent paragraphs depict dual sensitivity analysis scenarios and agreeing consequences in graphical procedures examined.

6.1 Scenario1: Sensitivity examination w. r. t. alteration in restoration rates**.** Taking e_i = 0.15 ($2 \leq i \leq 5$), a= 0.5, b=1 and variable α_i one by uniqueindividually at 0.55, 0.65, 0.75, 0.85

MTSF (T_0)

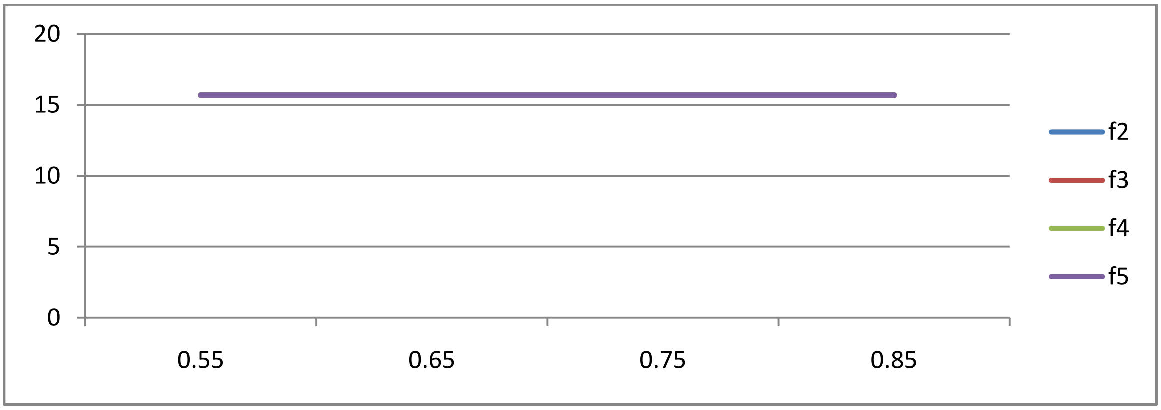

Fig. 2: MTSF

AOS (A_0)

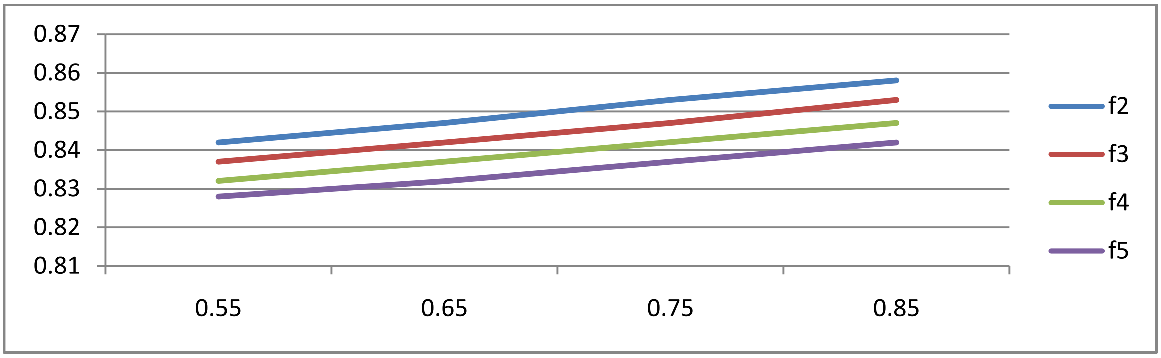

Fig. 3: AOS

SOBP (B_0)

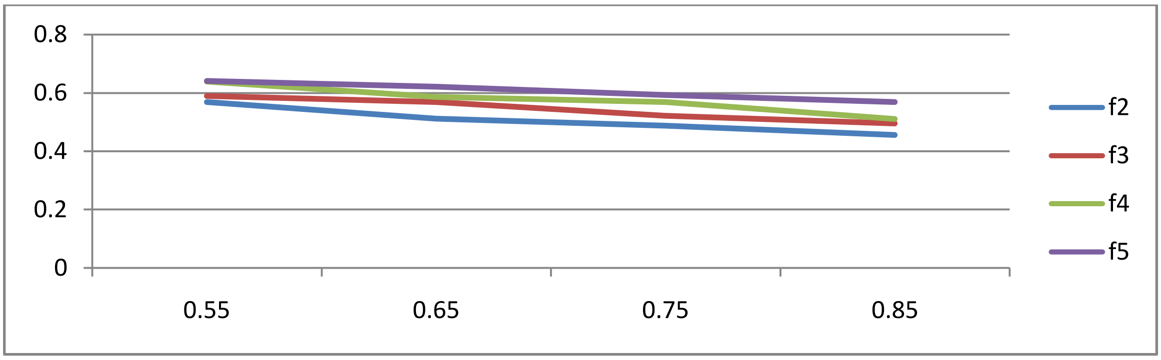

Fig. 4:SOBP

ENIR (V_0)

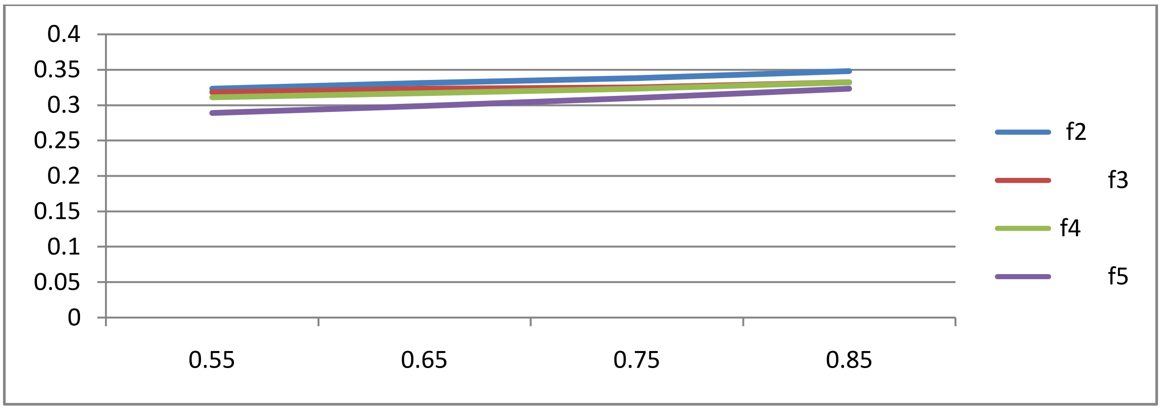

Fig. 5: ENIR

6.2 Scenario2: Currently one reflects sensitivity analysis scenario 2 for vagaries in failure rates: taking repair rate $f_i = 0.85$ $(2 \leq i \leq 5)$ and varying e_2, e_3, e_4, e_5 one by one correspondingly at 0.15, 0.25, 0.35, 0.45.

MTSF (T_0)

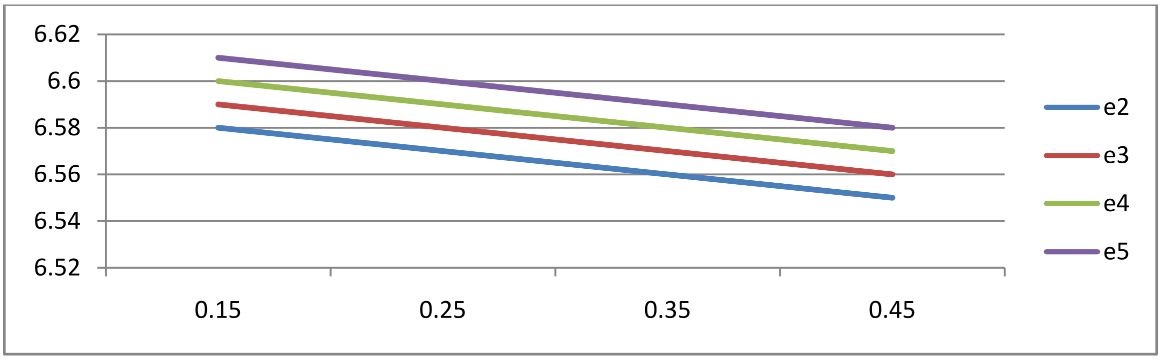

Fig. 6: MTSF

AOS(A_0)

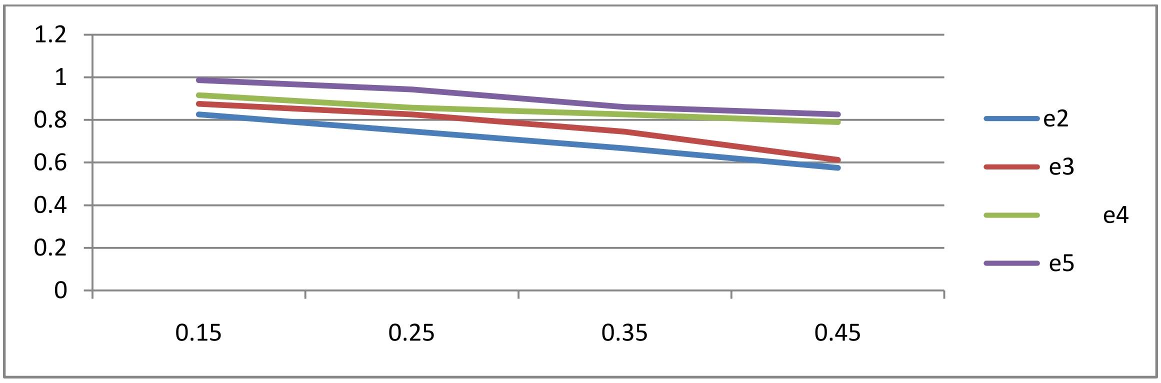

Fig. 7: AOS

SOBP (B_0)

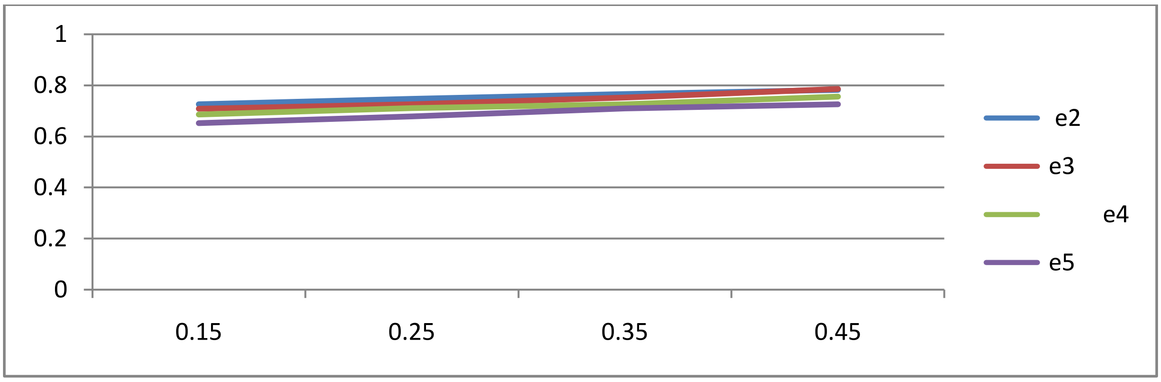

Fig. 8:SOBP

ENIR (V_0)

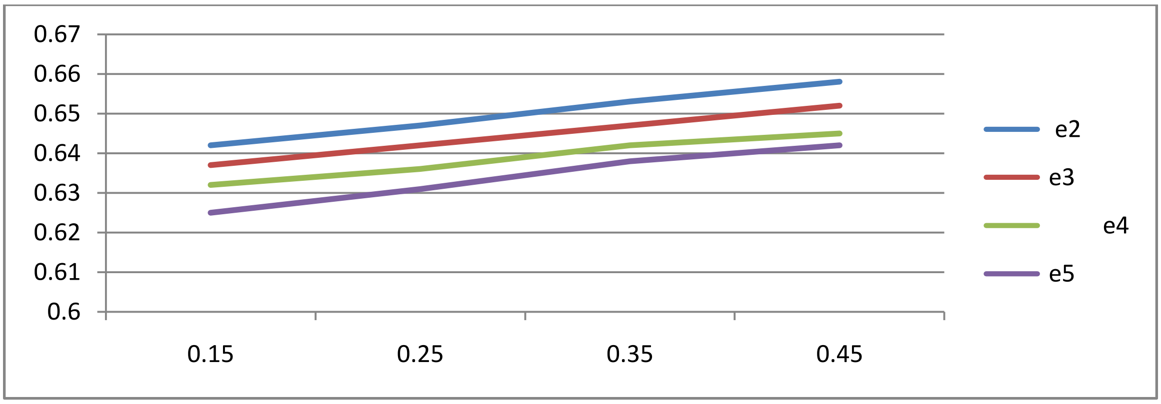

Fig. 9:ENIR

7. Conclusion

According to fig. 2, shows that the repair rates of individual units have no bearing on MTSF.Figure 3 come to the conclusion that an increase in reparation rates does not significantly rise the value of system accessibility. But the repair rate of unit "A" was kept at its highest level for maximum availability.It was determined in figure 4 that the busiest time for a server occurred when the repair rate of unit 'D' was at its highest level relative to other units. As a result, repairmen should be effective in fixing unit 'A' in command to decrease the busy time for the server.Conferring to Fig. 5, the number of calls from the server should be kept to a minimum while moving in segments from top to bottom. The estimation of V0 rises with the expansion of unit and server repair rates, but it rises more quickly with the relative increase in unit repair rates.Figure 6 demonstrates that as the failure rate of sub-units increases, estimation of T0 decreases more rapidly. As a result, unit "D" requires more cases for maintenance than other units, and diagram 6 illustrates this same trend.According to Fig. 7, availability is minimum when Unit 'A"s failure rate is 0.45 and its standards are 0.574, and accessibility is maximum when sub-units' disappointment rates are 0.15.From fig. 8, we understand that the best value of the SOBP is 0.782, which advises that the disappointment rate of unit is highest. After Fig. 9, it is determined that estimation of to rise at a quicker rate on expanding the disappointment rate of sub-unit rise, the same pattern is appreciated in figure 9.

References: -

1. Kumar, A., Goel, P., Garg, D., and Sahu, A. (2017). System behaviour analysis in the urea fertilizer industry. *Book: Data and Analysis [978-981-10-8526-0] Communications in computer and information Science (CCIS), Springer*, 3-12.https://doi.org/10.1007/978-981-10-8527-7_1
2. Rajbala, Kumar, A. and Khurana, P. (2022). Redundancy allocation problem: Jayfe cylinder Manufacturing Plant. *International Journal of Engineering, Science & Mathematic*, 11(1), 1-7.DOI:10.6084/m9.figshare.18972917.
3. Rajbala, Kumar, A. and Garg, D. (2019). Systems Modelling and Analysis: A Case Study of EAEP Manufacturing Plant. *International Journal of Advanced Science and Technology*, 28(14), 08-18.http://sersc.org/journals/index.php/IJAST/article/view/1419

4. Kumar, A., Goel, P. and Garg, D. (2018). Behaviour analysis of a bread making system. *International Journal of Statistics and Applied Mathematics*, 3(6), 56-61.
5. Kumar, A., Garg, D., Goel, P., Ozer, O. (2018). Sensitivity analysis of 3:4:: good system. *International Journal of Advance Research in Science and Engineering*, 7(2), 851-862.
6. Kumar, A., Garg, D., and Goel, P. (2019). Mathematical modelling and behavioural analysis of a washing unit in paper mill. *International Journal of System Assurance Engineering and Management*, 1(6), 1639-1645.DOI: 10.1007/s13198-019-00916-4
7. Kumar, A., Garg, D., and Goel, P. (2019). Sensitivity analysis of a cold standby system with priority for preventive maintenance. *Journal of Advance and Scholarly Researches in Allied Education*, 16(4), 253-258.http://ignited.in/I/a/88998
8. Kumar, A., Garg, D., and Goel, P. (2017). Mathematical modelling and profit analysis of an edible oil refinery industry. *Airo International Research journal*, XIII, 1-14.

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Performance Analysis of Cylinder Block in Cast Iron manufacturing plant

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Abstract: Complex repairable systems with more than two states—working, reduced, and failed—need to have their states appropriately modeled in order for the system to control a stochastic process while analyzing performance metrics. Five separate components make up a Cylinder Block in Cast Iron: a sand mixing unit, a sand core-making unit, a molding line, a sand extractor machine, and a fettling machine. The use of the time-homogeneous Markov process is utilized in this research to describe the dependability and availability of cylinder blocks in cast iron plants that involve reduced states. It is discovered to be an effective method that is entirely dependent on modeling and numerical analysis.

Keywords: Reliability, Cylinder Block in Cast Iron

1. Introduction

The pace of technological progress nowadays is quite quick, and every business strives to produce products that are more dependable and have longer lives and higher performance. Reliability is the likelihood of components, subassemblies, and manufacturing processes performing the required function for a desired time without failure in the specified situation with a perfect certainty. The likelihood that a component will not fail or require repair when it is needed is known as availability. It might seem logical that if a system is highly available, it should also be highly reliable. The practise of concentrating on lowering the breakdown by enhancing testability and adequate maintenance may also increase a component's availability. Proper maintenance planning and arranging is a crucial component.In this work, the dependability function and availability of cylinder blocks in a cast iron facility are examined to demonstrate the significance of the Markov process.

The availability function is provided by the state transition diagram for the system states, and the reliability function appears in the updated diagram. The expression for steady state availability is obtained when applying the normalising condition and is then optimised using a genetic algorithm. This study aims to (1) derive the reliability function and availability function from the Markov diagram of the cylinder block in the cast iron factory, and (2) construct a steady state availability optimization model with constant failure and repair rates for the components.

Kumar et al. [2018] analysed the behaviour of a bread plant.Kumar et al. [2019] used RPGT to perform a sensitivity analysis on a cold standby architecture composed of two identical units with a server failure and prioritised for preventative maintenance. The current paper is divided into two half, one of which is being used and the other of which is on cold standby. The main distinctions between online and cold standby equipment are the excellent and fully failed modes. Rajbala et al. [2019] studied a case study of an EAEP manufacturing facility in their work on system modelling and analysis in 2019. Kumar et al. [2017] investigated the behaviour of the urea fertiliser industry.

In 2017, Kumar et al. looked at the mathematical formulation and profit function of an edible oil refinery facility. Kumar et al. [2019] examined mathematical formulation and behaviour analysis in a paper mill washing unit. Agrawal et al. [2021] examined the Reverse Osmosis Water Treatment Plant using RPGT. In their research, Kumar et al. [2018] examined the sensitivity analysis of a 3:4:: remarkable system plant.

PSO was utilised by Kumari et al. [2021] to study certain circumstances. Rajbala et al. [2022] looked at the redundancy allocation issue in the cylinder manufacturing factory using a heuristic technique.

1.2 System Description:

Cylinder blocks fabricating plant including of 5 subsystems which are linked in series. The description of the subsystems is portrayed as base:

- Sand mixing machine (SMM) (P): cylinder blocks are primarily done using SMM unit. Balm and harder mix in silicon and to kind Sand unable aimed at core making.
- Sand core making machine (SCMM) (Q): In SCMM, blend sand and pour in cool box center making machine to make the SCMM.
- Moulding line machine (MLM) (R): In MLM, Sand core resolution in moulding line and molten cast iron pour in sand mould line.
- Sand extractor machine (SEM) (S): In SEM, Sand extract after casting by SEM.

- Fettling Machine (FM) (T): In FM, extra parts eliminate from casting, subsequently fettling casting communication to customer.

1.3 AssumptionsandNotations:

- P,Q,R,S,T: represents good working states.
- P,q,r,s,t: indicates failed states.
- α_i: indicates the failure rates.
- β_i: indicates the repair rates.
- Disappointment and fix rates are consistent.

1.4 State Conversion Diagram:

State conversion diagram 1 is used to address limited state machines.

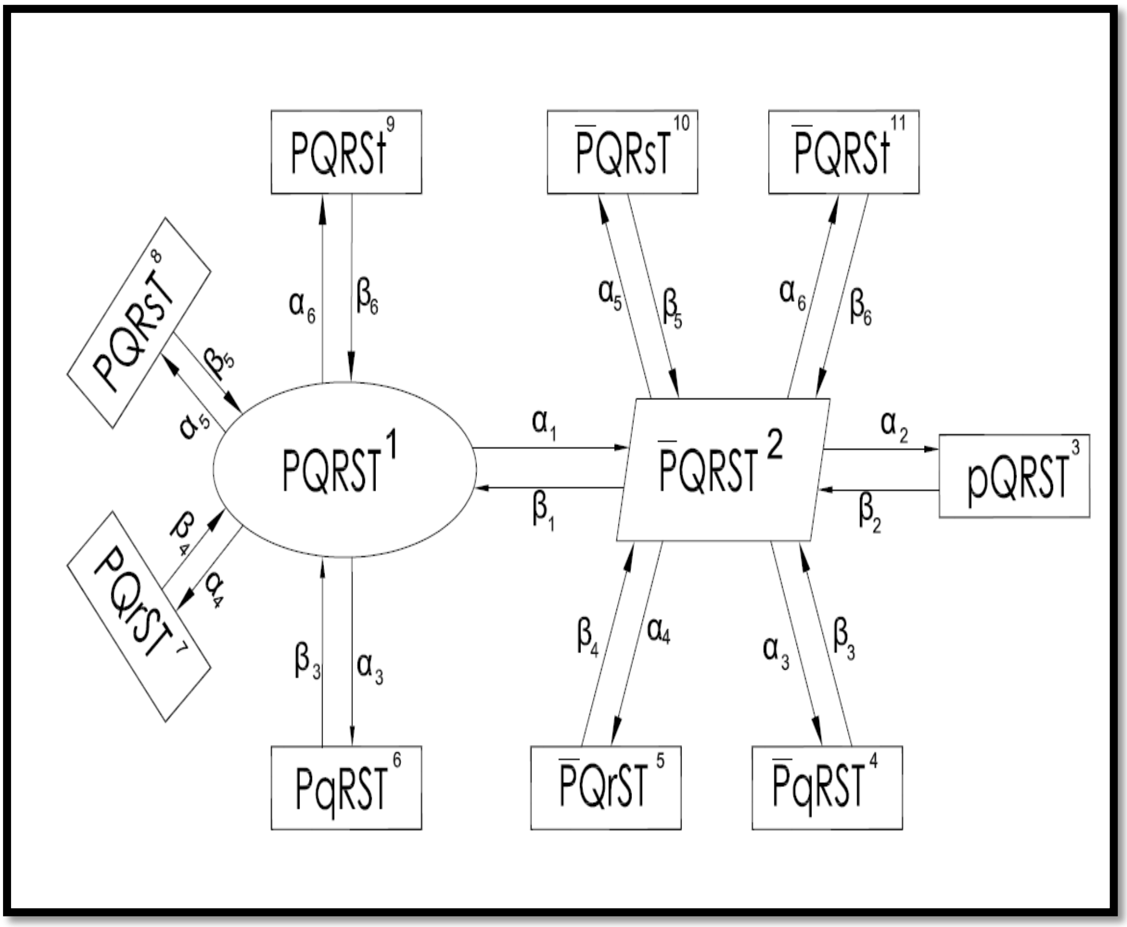

Fig. 1: Transition Diagram

2. **Results and Discussion:**

The following definitions describe the findings and discussions of the time-dependent availability, system reliability, and system steady-state availability:

Availability*:* It is the likelihood that a system will carry out its necessary function at a specific moment in time or over a predetermined amount of time when operated and maintained in accordance with guidelines.

Time dependent availability: It is distinct as the likelihood that the accessibility of the scheme is dependent upon the period.

2.1 Mathematical Modeling

Using the Marko birth-death process, the differential equations are governing the transition graphic. The following are the equations:

$p_1^{'}(t) + (\alpha_1 + \alpha_4 + \alpha_3 + \alpha_5 + \alpha_6)p_1(t) = \beta_1 p_2(t) + \beta_3 p_6(t) + \beta_4 p_7(t) + \beta_5 p_8(t) + \beta_6 p_9(t)$ (1)

$p'_2(t) + (\alpha_2 + \alpha_3 + \alpha_4 + \alpha_5 + \alpha_6 + \beta_1) p_2(t) = \beta_2 p_3(t) + \beta_3 p_4(t) + \beta_4 p_5(t) + \beta_5 p_{10}(t) + \beta_6 p_{11}(t) + \alpha_1 p_1(t)$ (2)

$p'_3(t) + \beta_2 p_2(t) = \alpha_2 p_2(t)$ (3)

$p'_4(t) + \beta_3 p_4(t) = \alpha_3 p_2(t)$ (4)

$p'_5(t) + \beta_4 p_5(t) = \alpha_4 p_2(t)$ (5)

$p'_{10}(t) + \beta_5 p_{10}(t) = \alpha_5 p_2(t)$ (6)

$p'_{11}(t) + \beta_6 p_{11}(t) = \alpha_6 p_2(t)$ (7)

$p'_6(t) + \beta_3 p_6(t) = \alpha_3 p_6(t)$ (8)

$p'_7(t) + \beta_4 p_7(t) = \alpha_4 p_7(t)$ (9)

$p'_8(t) + \beta_5 p_8(t) = \alpha_5 p_8(t)$ (10)

$p'_9(t) + \beta_6 p_9(t) = \alpha_6 p_9(t)$ (11)

Time dependent availability analysis (TDAA) by Runge- kutta 4th order technique in Matlab-tool

The algorithm below represents the general procedure for determining the value of TDAA:

Step 1

- Input values of period at which accessibility is to be designed i.e. Tspan = [0 20 40 60 80 100 120 140 160 180 200 220 240].
- Identify the initial circumstance as P (t = 0) = [1 0 0 0 0 0 0 0 0 0]

Step 2

- Set Scalar comparative error acceptance RelTol (1e-4) and a path of absolute error acceptance AbsTol (1e – 5).

Step 3

- Get the values of repair and disappointment α_i β_i where$1 \leq i \ \leq 6$. $\alpha_1 = 0.002$, $\alpha_2 = 0.003, \alpha_3 = 0.005$, $\alpha_4 = 0.007$, $\alpha_5 = 0.009$, $\alpha_6 = 0.011$. $\beta_1 = 0.02$, $\beta_2 = 0.03$, $\beta_3 = 0.05$, $\beta_4 = 0.07$, $\beta_5 = 0.09$, $\beta_6 = 0.11$.
- Input the linear DE after (ldefun) 3.1 to 3.11.

Step 4

- Call ode45 (with ldefun, tspan, p (t = 0) as input dispute).

Step 5

- Give the sum of operational states $P_1(t)$, $P_2(t)$.

$$A_v(t) = P_1(t) + P_2(t)$$

Step 6

- Print the value of accessibility consistent to different period value in Tspan. Stop the algorithm.

A matlab program is established to total the consequences.

Table1: Effect of availability w.r.t. time

Time (months)	Availability
0	1
15	0.93
30	0.89
45	0.81
60	0.77
75	0.71
90	0.67
105	0.58
120	0.47
135	0.42
150	0.38
165	0.33
180	0.25
195	0.20
210	0.13

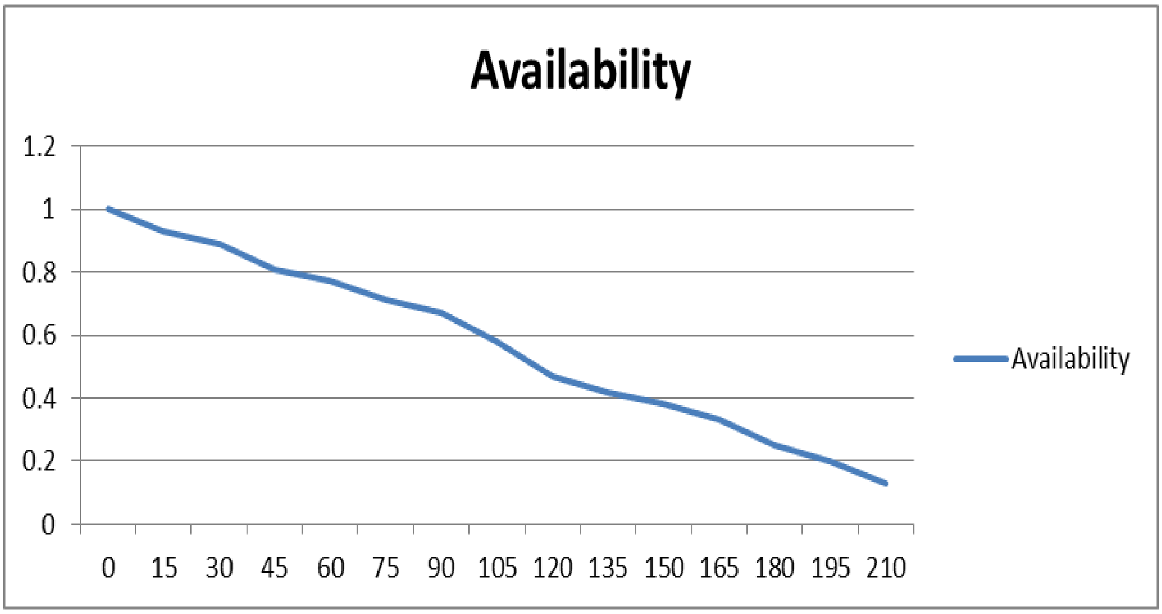

Fig. 2: Variation of availability w.r.t. time

Maintenance Precedence

In order to maintain the quality of the machinery and the finished product, industrial maintenance must be planned ahead of time. Nowadays, planners and schedulers can execute their duties more quickly and effectively with the help of computer applications. For instance, one such tool can allocate the appropriate personnel and resources for maintenance work based on the priority of the work and the equipment's availability. An organization's maintenance procedure must follow its policies, as well as its standards for worker safety, regulatory compliance, and asset-driven requirements. The majority of these needs can be satisfied via preventive maintenance methods.

Preventative Maintenance, also known as Planned Maintenance, aims to avoid major failures using a well-defined, regular maintenance programmer. Moreover, it is utilized to replace consumables like oil. The drawback of preventive maintenance is that when parts wear out, they may need to be replaced, and there is always a chance that the repair process itself will cause additional damage. These are the subsystems' maintenance priorities based on repair rates:

Table 2: Maintenance precedence of the subsystems based on repair rates

Subsystem names	Increase availability	Rank
P	27.35%	1st
Q	23.69%	2nd
R	20.08%	3rd
S	19.78%	4th
T	17.53%	5th

3. Conclusion

When it comes to maintenance, subsystem P is clearly the most important subsystem. Subsystem P should therefore be given priority because it has a substantially higher repair rate on availability than other subsystems. Table 1 indicates that the system's time-dependent availability decreases over time.

References: -

1. Rajbala, Kumar, A. and Garg,, D., *Systems Modeling and Analysis: A Case Study of EAEP Manufacturing Plant,* International Journal of Advanced Science and Technology, vol. 28(14), 2019, 08-18.
2. Agrawal, A., Garg, D., Kumar, A. and Kumar, R., *Performance Analysis of the Water Treatment Reverse Osmosis Plant,* Reliability: Theory & Applications, 2021, 16-25.
3. Kumar, A., Garg, D., and Goel, P., *Mathematical modeling and behavioral analysis of a washing unit in paper mill*, International Journal of System Assurance Engineering and Management, vol. 1(6), 2019, 1639-1645.
4. Kumar, A., Garg, D., and Goel, P., *Sensitivity analysis of a cold standby system with priority for preventive maintenance*, Journal of Advance and Scholarly Researches in Allied Education, vol. 16(4), 2019, 253-258.
5. Kumar, A., Goel, P. and Garg, D., *Behaviour analysis of a bread making system,* International Journal of Statistics and Applied Mathematics, 3(6), 2018, 56-61.
6. Kumar, A., Garg, D., Goel, P., Ozer, O., *Sensitivity analysis of 3:4:: good system,* International Journal of Advance Research in Science and Engineering, 7(2), 218, 851-862.
7. Kumar, A., Garg, D., and Goel, P., *Mathematical modeling and profit analysis of an edible oil refinery industry*, Airo International Research journal, XIII, 2017, 1-14.
8. Kumar, A., Goel, P., Garg, D., and Sahu, A., *System behavior analysis in the urea fertilizer industry*, Book: Data and Analysis [978-981-10-8526-0] Communications in computer and information Science (CCIS), Springer, 2017, 3-12.
9. Kumari, S., Khurana, P., Singla, S., Kumar, A., *Solution of constrained problems using particle swarm optimization,* International Journal of System Assurance Engineering and Management, 2021, pp. 1-8.
10. Rajbala, Kumar, A. and Khurana, P., *Redundancy allocation problem: Jayfe cylinder Manufacturing Plant.* International Journal of Engineering, Science & Mathematic, vol. 11, issue 1, 2022, 1-7.DOI:10.6084/m9.figshare.18972917.

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MATHEMATICAL MODELING AND PROFIT ANALYSIS OF A SOAP INDUSTRY

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ABSTRACT: This article uses RPGT to analyse the profitability of the soap industry. Three non-identical units make up the dependability model for accessibility of the soap industry, where the main unit is capable of operating in a reduced state in the event of a partial failure. A partial failure of the main unit can result in an upstate, a partially failed state, or a wholly failed state.

In a partially unsuccessful state, the system can still function with a decreased capacity. The unit's repair, the server's treatment, and the switch devices' care are all regarded as excellent.The MTSF, server of busy period and the number of attendant visits have all been appraised using the RPGT.

Keywords: RPGT, busy period of the server, MTSF

1. INTRODUCTION:

Reliability recital measures must incredible portion in modern scheme, Soap industry, bread manufacture system, control plants, business systems and to keep up more significant level or edge level of a reliability amount require in the agenda. In the popular of the schemes these stages are reserved up by philanthropic proficient reparation and keep activities, yet at the same time to save highest level standby redundant units for an exact system are admired. In the initial step of the soap-making process, fats then alkali remainliquefied in a pan, a steel cistern that may support several thousand pounds and stand three stories tall.

Following boiling, the mixture thickens as soap and glycerin are created when the fat and alkali react.

Removing sodium chloride, sodium hydroxide, and glycerol from soap is a common industrial technique. By re-precipitating the soap per salt next the crude soap curds have been boiled in water to eliminate these components. The aquatic is then taken after the soap following this. The now-dry soap is next flattened into tiny bitsbya moisturehappy of crudely 5-13%.These pellets are now prepared for finishing with soap. Rare soap pellets are transformed into a salableinvention, typically bars, through the finishing process. In an amalgamator, soap balls are sundrythrutracks and other ingredients until they are homogeneous. The build is then removed as of the mixer and poured into a refiner, where a thin wire screen is used to filter the soap using an auger. Alike to calendaring paper or else plastic or creating chocolate whiskey, soap from the refiner is fed into a wave mill.

To further plasticize the soap mass, one or more refiners are then run through the soap. It goes through a vacuum chamber right before extrusion in order to release any trapped air.It is then cut to useful lengths, extruded into a long log or blank, put done a metal indicator, besidesimprinted with cold-tools addicted to the desired shape. The pressed bars are then offered in a variety of packages. The following system attributes have been assessed using the RPGT in order to analyze the performance of the arrangement.

Shakuntla et al [2011] discussed the comportment analysis of polytube using supplementary variable technique the behavior of a bread plant was examined by Kumar et al. in [2018]. In order to do a sensitivity analysis on a cold standby framework made up of two identical units with server failure and prioritized for preventative maintenance, Kumar et al. [2019] used RPGT. Two halves make up the current paper, one of which is in use and the other of which is in cold standby mode. In their study, Kumar et al. [2018] investigated a 3:4:: outstanding system plant's sensitivity analysis. PSO was used by Kumari et al. [2021] to research limited situations.

Using a heuristic approach, Rajbala et al. [2022] investigated the redundancy allocation problem in the cylinder manufacturing plant.The good and fully failed modes are the only differences between online and cold standby equipment. A study of the urea fertilizer industry's behavior was conducted by Kumar et al. [2017]. Accurate formulation and profit function of a comestible oil refinery facility were investigated by Kumar et al. in [2017]. In a paper mill washing unit, Kumar et al. [2019] investigated mathematical formulation and behavior study. A case study of an EAEP manufacturing facility was examined by Rajbala et al. [2019] in their work on system modeling and analysis in [2019]. The comparative analysis of the subsystem failed simultaneously was discussed by Shakuntla et al. [2011].

2. ASSUMPTIONS AND NOTATIONS:

- A separate repair facility is available 24x7 hrs.
- Repairs of units are perfect.
- Repaired unit works as if it is a new.

$\bar{A}$: partial failed state

a :failed state

A :Good state

3. TRANSITION DIAGRAM

Resulting the upstairsexpectations and notations, the transition illustration of the organization is revealed in fig.1

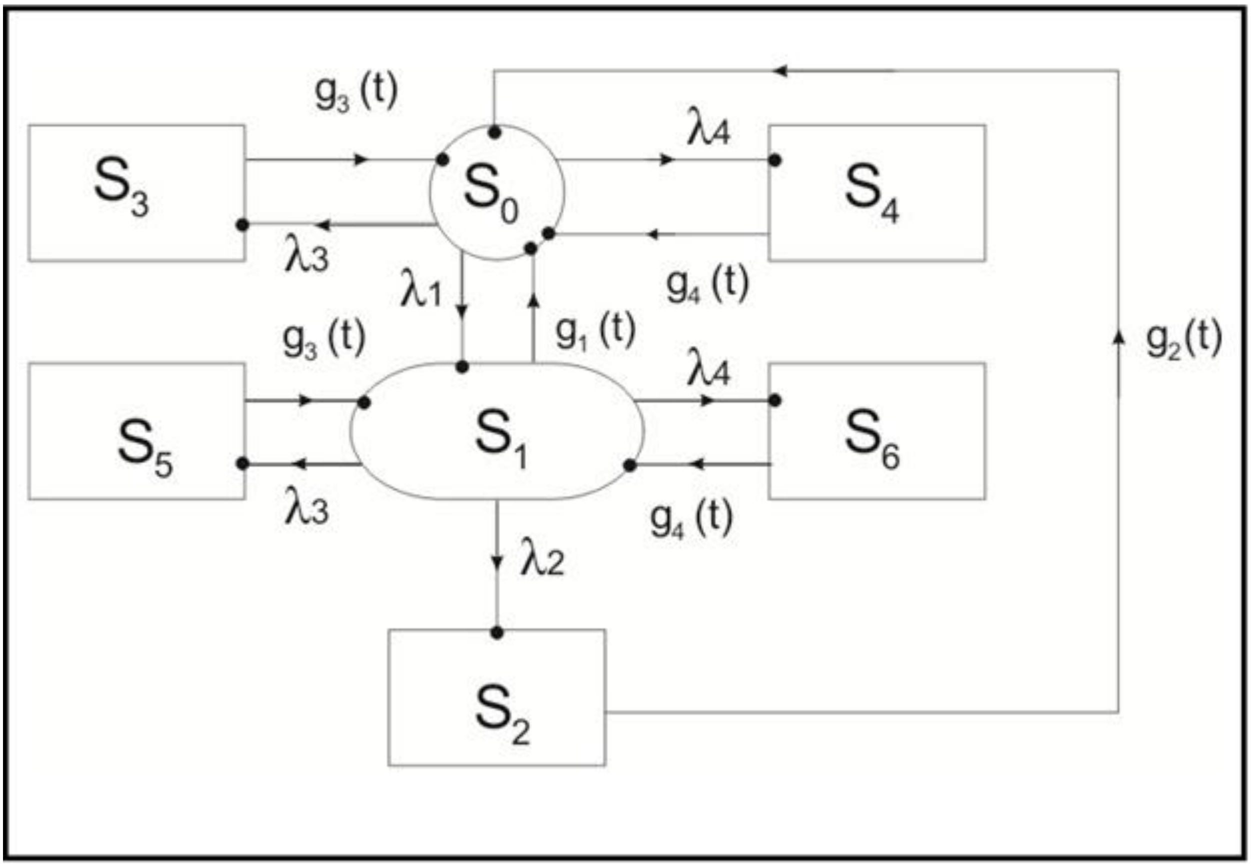

Figure 1: Transition Diagram

S_0 = ABC; S_1= ĀBC; S_2 = aBC; S_3 = AbC; S_4 =ABc; S_5 = ĀbC; S_6 = ĀBc

4. TRANSITION PROBABILITIES AND MEAN SOJOURN TIMES:

$q_{i,j}(t)$: Probability supply utility after i to j. pi,j: Transition likelihood after i to j.

$p_{i,j} = q_{i,j}{*}(0)$; where * entitle Laplace transform.

Table 1: Transition Probabilities

$q_{i,j}(t)$	$p_{i,j}=q_{i,j}{*}(0)$
$q_{0,1}= \lambda_i\, e^{-(\lambda 3+\lambda 1+\lambda 4)t}$ i = 1,3,4	$p_{0,i}= \lambda_1/ \lambda_3+ \lambda_1+ \lambda_4$ i = 1,3,4
$q_{1,0}=g_1(t)e^{-(\lambda 4+\lambda 3+\lambda 2)t}$ $q_{1,i}= \lambda_j e^{-(\lambda 4+\lambda 3+\lambda 2)t}\, \bar{G}_1(t)$ i = 2, 5, 6 j = 2, 3, 4	$p_{1,0}=g_1{*}(\lambda_4+ \lambda_3+ \lambda_2)$ $p_{1,2}= \lambda_2[1-g_1{*}(\lambda_4+ \lambda_2+ \lambda_4)]/ \lambda_4+ \lambda_3+ \lambda_2$ i = 2, 5, 6 j = 2, 3, 4
$q_{2,0}(t)=g_2(t)$	$p_{2,0}= 1$
$q_{3,0}(t)=g_3(t)$	$p_{3,0}=1$
$q_{4,0}(t)=g_4(t)$	$p_{4,0}=1$
$q_{5,1}(t)=g_3(t)$	$p_{5,1}=1$
$q_{6,1}(t)=g_4(t)$	$p_{6,1}=1$

Table 2: Average so-journal time

$R_i(t)$	$\mu_i=R_i{*}(0)$
$R_i(t)= e^{-(\lambda 4+\lambda 3+\lambda 1)t}$	$\mu_0=1/ \lambda_4+ \lambda_3+ \lambda_1$
$R_1(t)= e^{-(\lambda 4+\lambda 3+\lambda 2)t}\, \bar{G}_1(t)$	$\mu_1=1- g_1{*}(\lambda_4+ \lambda_3+ \lambda_2)/ \lambda_4+ \lambda_3+ \lambda_2$
$R_2(t)= \bar{G}_2(t)$	$\mu_2= - g_2{*}'(0)$
$R_3(t)= \bar{G}_3(t)$	$\mu_3= - g_3{*}'(0)$
$R_4(t)= \bar{G}_4(t)$	$\mu_4= - g_4{*}'(0)$
$R_5(t)= \bar{G}_3(t)$	$\mu_5= - g_3{*}'(0)$
$R_6(t)= \bar{G}_4(t)$	$\mu_6= - g_4{*}'(0)$

5. Evaluation of Path Likelihoods

Smearing RPGT and thru '0' as initial-state of framework as beneath: The transition likelihood factors of completelyreachable states after base state 'ξ' = '0' remain:

$V_{0,0}$ =1 (Verified)

$V0,1=[p_{0,1}/\{1-(p_{1,5}p_{5,1})\}\{1-(p_{1,6}p_{6,1})\}]$

$V0,2=[p_{0,1}\ p_{1,2}/\{1-(p_{1,5}p_{5,1})\}\{1-(p_{1,6}p_{6,1})\}]$

$V0,3= p_{0,3}$

$V0,4= p_{0,4}$

$V0,5=[p_{0,1}\ p_{1,5}/\{1-(p_{1,5}p_{5,1})\}\{1-(p_{1,6}p_{6,1})\}]$

$V0,6 =[p_{0,1}\ p_{1,6}/\{1-(p_{1,5}p_{5,1})\}\{1-(p_{1,6}p_{6,1})\}]$

(a). MTSF(T_0): Re-forming un-failed positions to which the context container shipment, afore inflowing one vain state are: 'i' = 0, 1.

$$\text{MTSF}\ (T_0) = \left[\sum_{i,sr}\left\{\frac{\left\{pr\left(\xi \xrightarrow{sr\ (sff)} i\right)\right\}\mu i}{\Pi_{m_{1\neq\xi}}\{1-V_{\overline{m_1 m_1}}\}}\right\}\right] \div \left[1-\sum_{sr}\left\{\frac{\left\{pr\left(\xi \xrightarrow{sr\ (sff)} \xi\right)\right\}}{\Pi_{m_{2\neq\xi}}\{1-V_{\overline{m_2 m_2}}\}}\right\}\right]$$

$\text{MTSF} =[\mu_0+p_{0,1}\mu_1] \div [1- \{(\ p_{0,1}\ p_{1,0})\}]$

(b). Availability (A_0): The organisation is presented when 'j' = 0, 1 taking 'ξ' = '0' the total fraction of periodon behalf of which the organisation is presented is assumed by":

$$A_0= \left[\sum_{j,sr}\left\{\frac{\{pr\,(\xi^{sr}\rightarrow j)\}fj,\mu j}{\Pi_{m_{1\neq\xi}}\{1-V_{\overline{m_1 m_1}}\}}\right\}\right] \div \left[\sum_{i,s_r}\left\{\frac{\{pr\,(\xi^{sr}\rightarrow i)\}\mu_i^1}{\Pi_{m_{2\neq\xi}}\{1-V_{\overline{m_2 m_2}}\}}\right\}\right]$$

$A_0 = [V_{0,0}f_0\ \mu_0+ V_{0,1}f_1\ \mu_1] \div [V_{0,0}\ \mu_0^1 V_{0,1}\mu_0^1+ V_{0,2}\ \mu_2^1+ V_{0,3}\ \mu_3^1+ V_{0,4}\mu_4^1+ V_{0,5}\ \mu_5^1+ V_{0,6}\ \mu_6^1]$

After putting particular values to repair and failure rates, we get the variation of availability of system for a constant state independent of time.

(c). Busy period of Server (B_0): The attendant is busy in all states except the initial state the over-all fraction of periodon behalf of which the attendant remains demanding is:

$$B_0 = \left[\sum_j V_{\xi,j}\ , n_j\right] \div \left[\sum_i V_{\xi,i}\ , \mu_i^1\right]$$

$B_0 = [V_{0,1}\ \eta_1 + V_{0,2}\ \eta_2 + V_{0,3}\ \eta_3 + V_{0,4}\ \eta_4 + V_{0,5}\ \eta_5 + V_{0,6}\ \eta_6] \div [V_{0,0}\ \mu_0^1 + V_{0,1}\ \mu_1^1 + V_{0,2}\ \mu_2^1 + V_{0,3}\ \mu_3^1 + V_{0,4}\ \mu_4^1 + V_{0,5}\ \mu_5^1 + V_{0,6}\ \mu_6^1]$

(d)Expected number of Server's visits(V_0): The states where the repairman ensureappointment's a fresh are j= 1, 3, 4taking 'ξ' = '0',

$V_0 = \left[\sum_j V_{\xi,j}\right] \div \left[\sum_i V_{\xi,i}\,, \mu_i^1\right]$

$V_0 = [V_{0,1} + V_{0,3} + V_{0,4}] \div [V_{0,0}\ \mu_0 + V_{0,1}\ \mu_1 + V_{0,3}\ \mu_3 + V_{0,4}\ \mu_4 + V_{0,5}\ \mu_5 + V_{0,6}\ \mu_6]$

6. Profit Function (PF):

The PF of the organization can be completed by expending the PF:

$P_0 = E_1A_0 - E_2B_0 - E_3V_0$

Where;

$E_1 = 500$

$E_2 = 100$

$E_3 = 200$

$\lambda_1 = \lambda_2 = \lambda_3 = \lambda_4 = \lambda$, $\omega_1 = \omega_2 = \omega_3 = \omega_4 = \omega$

Table 3: PF

λ \ w	.55	.65	.75
.15	185	252	263
.25	71	82	105
.35	44	52	63

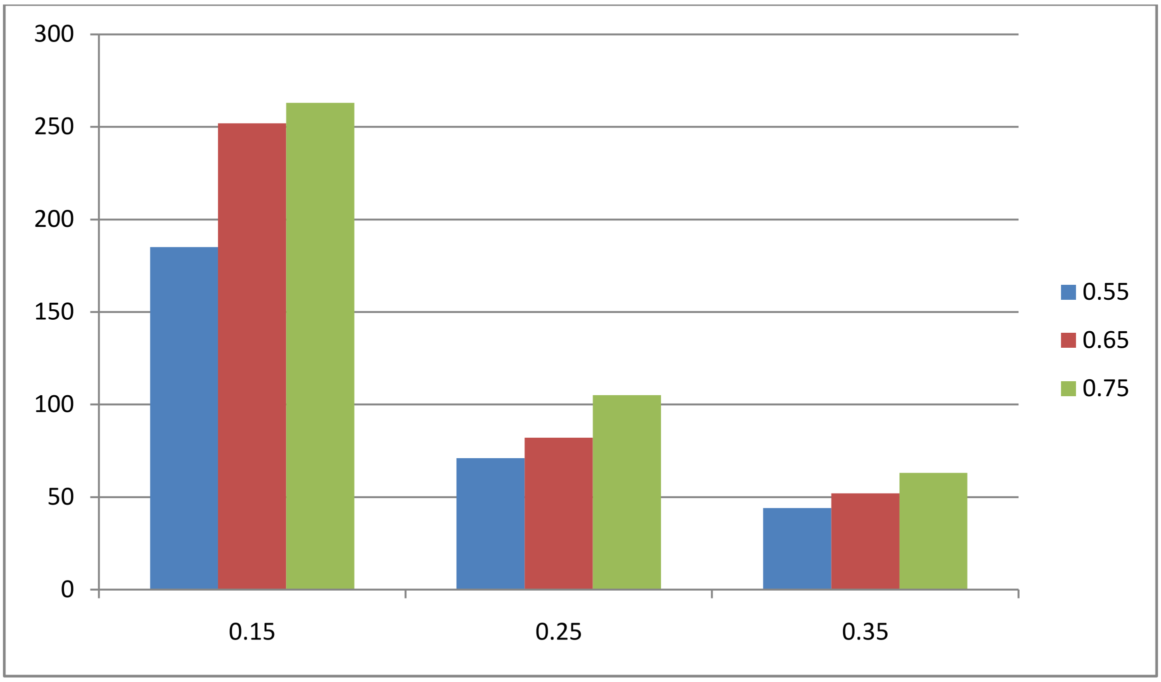

Fig. 2: PF

Situating the found values for P_0, profit table 3 and graph 2 may besides be set and conclusion with respect to repair and disappointment rates of units.

7. CONCLUSION:

From the analytical and figure discussions, it is noted availability of the system, profit function, anticipated number of check-ups by repairman are decremented with increase disappointment rate and they all rise as the repair rate increases. Busy period of server and MTSF are decremented with increase repair rates. The efficiency and the consistency of the herbal can be better-quality by snowballing reparation rate and declining the disappointment rate.

8. References: -

1. Kumar, A., Garg, D., and Goel, P. (2017), "Mathematical modelling and profit analysis of an edible oil refinery industry", Airo International Research journal, XIII, 1-14.
2. Kumar, A., Goel, P., Garg, D., and Sahu, A. (2017)," System behaviour analysis in the urea fertilizer industry", Book: Data and Analysis [978-981-10-8526-0] Communications
3. in computer and information Science (CCIS), Springer, 3-12.

4. Kumari, S., Khurana, P., Singla, S., Kumar, A. (2021) Solution of constrained problems using particle swarm optimization, International Journal of System Assurance Engineering and Management, pp. 1-8.
5. Kumar, A., Garg, D., and Goel, P. (2019), "Sensitivity analysis of a cold standby system with priority for preventive maintenance", Journal of Advance and Scholarly Researches in Allied Education, 16(4), 253-258.
6. Shakuntla ,Lal, A,K., and Bhatia, S.S., (2011), "Reliability analysis of polytube tube industry using supplementary variable Technique".Applied Mathematics and Computation.281, 3981-3992.
7. Kumar, A., Goel, P. and Garg, D. (2018), "Behaviour analysis of a bread making system", International Journal of Statistics and Applied Mathematics, 3(6), 56-61.
8. Kumar, A., Garg, D., Goel, P., Ozer, O. (2018), "Sensitivity analysis of 3:4:: good system", International Journal of Advance Research in Science and Engineering, 7(2), 851-862.
9. Rajbala, Kumar, A. and Khurana, P. (2022). Redundancy allocation problem: Jayfe cylinder Manufacturing Plant. International Journal of Engineering, Science & Mathematic, vol. 11, issue 1, 1-7.DOI:10.6084/m9.figshare.18972917.
10. Rajbala, Arun Kumar and DeepikaGarg, (2019) "Systems Modelling and Analysis: A Case Study of EAEP Manufacturing Plant", International Journal of Advanced Science and Technology, vol. 28(14), pp 08-18, 2019.
11. Shakuntla, Lal, A, K., and Bhatia, S.S., (2011) "Comparative study of the subsystems subjected to independent and simultaneous failure", EksploatacjaINiezawodnosc-Maintenance and Reliability. 4, 63-71.
12. Kumar, A., Garg, D., and Goel, P. (2019), "Mathematical modelling and behavioural analysis of a washing unit in paper mill", International Journal of System Assurance Engineering and Management, 1(6), 1639-1645.

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An Estimation of the Modified Maximum Likelihood for the Three Parameters in the Lognormal Distribution Model

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Abstract:-When we try to estimate these parameters using a certain sample, the complexities that are caused by the introduction of the threshold limits in the 3 parameter lognormal circulation($\boldsymbol{\lambda}$, μ, σ) are created. The possibility function of every ordered examplex_1 ,........., $x_n$goes to infinity when ($\boldsymbol{\lambda}$, μ, σ) approach ($x_1, -\infty, \infty$)accordingly, which caused global maximum likelihood estimation to provide inadmissible estimates. In this research, we present a new modified form of maximum likelihood estimation to adapt to the problem by employing Quasi-Newton Raphson iteration to estimate parameter. This version of maximum likelihood estimation uses a modified version of maximum likelihood estimation. A stringent simulation test was used to evaluate both the performance of the new method as well as the performance of the existing method for estimating the local maximum likelihood. An investigation into the behaviour of the estimators in relation to the asymptotic normality assumption was carried out in the form of a sensitivity analysis. In order to provide an explanation of how the suggested strategy works, it was applied to an actual data set consisting of accident claim costs from a reputable insurance company in Malaysia.

Keywords: -Modified Maximum Likelihood,Quasi-Newton Raphson, Logarithmic transformation

1. Introduction

A random flexible Y is assumed to must a lognormal distribution by parameters μ and 2 σ , if lnY is distributed as normal. In this notation, μ is a scale stricture and σ is anoutline parameter. The probability density function for two parameters lognormal distribution can be written as:

$$f(\ln y;\mu,\sigma^2)=\begin{cases}\frac{1}{\sqrt{2\pi\sigma^2}\mathrm{y}}\exp\left\{-\frac{(\ln y-\mu)^2}{2\sigma^2}\right\}, & ,\mathrm{y}>0,\sigma^2>0,-\infty<\mu<\infty \\ 0 & ,\mathrm{y}\le 0\end{cases} \quad (1)$$

By assuming that, a random variable X =ln(Y −λ) with λ> 0 is similarly distributed as normal with parameters μ and 2 σ . Accordingly, the pdf of X can be carved as

$$f(x;\lambda,\mu,\sigma^2)=\begin{cases}\frac{1}{\sqrt{2\pi\sigma^2(x-\lambda)}}\exp\frac{-1}{2\sigma^2}[\ln(x-\lambda)-\mu]^2, & \lambda<\mathrm{x}<\infty,\sigma^2>0, \\ 0 & -\infty<\mu<\infty \\ \text{elsewhere} & \end{cases} \quad (2)$$

In this paper we modified the existing estimation method of the three parameters of lognormal distribution. The idea of this method has come out from the problem stated by Hill (1963) as in who showed that global maximum likelihood estimators lead to inadmissible estimates as the likelihood function of onemethodical sample x_1 ….. x_n tends to infinity when (λ, μ,σ) line(x_1 ,–∞,∞) 1 x respectively.Since of the status of the 3 parameters lognormal dispersal, there are all-embracing literatures on the valuationsystemsaimed at this distribution. Seufert (2021) Due to the fact that investigations into subjectivity Quality of Experience (QoE) utilisebiassed assumptions regarding ordinal rating scale, Mean Percentage Score (MOS)-based assessments provide results that are problematic and misleading. Mishra et al. (2019) Epidemiological, statistical approaches for data processing and categorization are accessible. These methodologies can be applied to each and every specific circumstance. We went through parametric and non-parametric methodologies, their prerequisites, and how to choose appropriate statistical measurement and analysis, as well as interpretation of biological data, in this post. The Senators Sarmento and Costa (2019) The application of statistical software in both business and academic settings has become increasingly common during the past few years. Everyone from students and professors to experts and average users has had some experience with statistical software at some time in their lives. In this study, we make an effort to make access to various theoretical concepts easier by providing a statistical review of such concepts. Peligrad (2018) conducted research on the exact restrained and large unorthodoxy asymptotic in non-logarithmic technique for linear methods that have self-governing innovations. Since the linear processes that we investigate are universal in nature, we will refer to them as the "long memory case." Savsani and Ghosh (2017) in this article, we have developed a method for finding the posterior distribution of the

Moderate Distribution by making use of the likelihood of a single observation and its ordinates. This method was inspired by Savsani and Ghosh's 2017 study.

Distributions such as exponential (Kiefer, 1984)and Weibull (Faveroet. al, 1994), which are also subfamilies of GGD, are utilized in the process of duration analysis in economics. Additionally, lognormal, which is considered to be a limiting spreading, consumes been utilized in finances by Jasggia (1991).They modified the differentiation equation of parameter λ by replacing $E\big(R(X_r)\big) = F(X_r)$and in$(x_r - \lambda) = \mathrm{E}(Z_r)$,where X_r is the sample value. Lawrence (1979, 1980, 1984) as in applied the three restriction lognormal dispersal in marketing and sociological studies.

2. LOCAL MAXIMUM LIKELIHOOD ESTIMATION (LMLE)

The estimators of the three strange parameters $(\lambda, \mu, \sigma^2)$for lognormal distribution can be obtained by using the following procedure:

Step 1: Get the likelihood utility of the circulation,

$$L(\lambda, \mu, \sigma^2) = \frac{1}{(2\pi\sigma^2)^{\sigma/2}} \prod_{i=1}^{n} \frac{1}{(x_i - \lambda)} \exp\frac{-1}{2\sigma^2} \sum_{i=1}^{n} [\ln(x_i - \lambda) - \mu]^2 \tag{3}$$

Step 2: Take the normal log of the likelihood function obtained in Step 1 above

$$\ln L(\lambda, \mu, \sigma^2) = -\frac{n}{2}\ln(2\pi) - n\ln(\sigma) - \sum_{i=1}^{n} \ln(x_i - \lambda) - \frac{1}{2}\sigma^{-2} \sum_{i=1}^{n} [\ln(x_t - \lambda) - \mu]^2 \tag{4}$$

Step 3: Find the partial offshoots of log-likelihood function:

$$\frac{\partial \ln L}{\partial \mu} = \frac{1}{\sigma^2} \sum_{i=1}^{n} [\ln(x_i - \lambda) - \mu] \tag{5}$$

$$\frac{\partial \ln L}{\partial \sigma} = -\frac{n}{\sigma} + \frac{1}{\sigma^3} \sum_{i=1}^{n} [\ln(x_i - \lambda) - \mu]^2 \tag{6}$$

$$\frac{\partial \ln L}{\partial \lambda} = \frac{1}{\sigma^2} \sum_{i=1}^{n} \frac{[\ln(x_i - \lambda) - \mu]}{x_i - \lambda} + \sum_{1}^{n} (x_i - \lambda)^{-1} \tag{7}$$

Step 4: Equate the derivatives in Step 3 with zero and solve Equation (5) and (6) for μ and σ^2.

$$\hat{\mu} = \frac{1}{n} \sum_{i=1}^{n} \ln\big(x_i - \hat{\lambda}\big) \tag{8}$$

$$\hat{\sigma}^2 = \frac{1}{n} \sum_{i=1}^{n} \big[\ln\big(x_i - \hat{\lambda}\big) - \hat{\mu}\big]^2 \tag{9}$$

Substitute μ and σ^2 into (7) and equating with zero,

$$g(\hat{\lambda}) = n \sum_{i=1}^{n} \big[\ln(x_i - \hat{\lambda}) - \hat{\mu}\big]^{-2} \times \sum_{i=1}^{n} \frac{\big[n\ln(x_i - \lambda) - \sum_{i=1}^{n} \ln(x_i - \hat{\lambda})\big]}{n\,(x_i - \lambda)} + \sum_{i=1}^{n} (x_i - \lambda)^{-1} = 0 \tag{10}$$

Equation (10) can be solved iteratively by the method of Newton Raphson algorithm. Clearly, from Equation (10) it shows that g:λ→ as λ→-∞, but λ=-∞ obviously is not a reasonable value for an estimator. Therefore, during the iteration process, $\hat{\lambda}$ has to be blocked from going to -∞ Similarly, if (1) λ→ x (where (1) x is the smallest observation), then g(λ)→∞ .

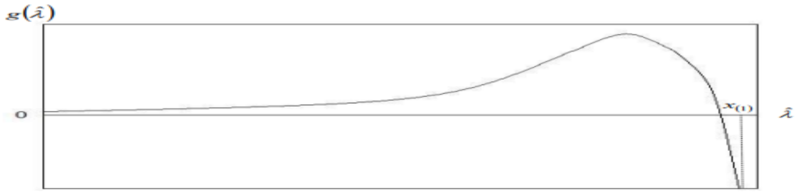

Figure 1: Scratch graph of $(\hat{\lambda})$ g against $\hat{\lambda}$

3. MODIFIED MAXIMUM LIKELIHOOD ESTIMATION (MMLE)

Rather than using the Newton Raphson iteration, we limit consideration to modified maximum likelihood estimation that use (1)$\hat{\lambda}$= x_1 the smallest value of sample x, and we make some modification to the equation (8) and (9) such that:

$$\hat{\mu} = \frac{1}{n}\sum_{i=2}^{n} \ln(x_i - \hat{\lambda}) \tag{11}$$

and$$\hat{\sigma}^2 = \frac{1}{n}\sum_{i=2}^{n} \left[\ln(x_i - \hat{\lambda}) - \hat{\mu}\right]^2 \tag{12}$$

The above equations differ from LMLE, such that the first value of summation is considered from the second value of ordered sample.

This method can solve two problems of likelihood function:

The value of any ordered sample x_1……..,x_n will not tends to ∞ as (λ ,μ, σ^2) approach (x_1–∞, ∞). This method can be used for any sample sizes. It avoids the problem of convergence in Newton-Raphson iteration method which produces is no solutions for λ when sample size is very small (n < 10) as shown by Colin Chen (2005). However, this method has some disadvantages. Some information about the sample is lost since the smallest value of x is not included in the formulation to calculate the estimate values of μ and σ^2.

4. ASYMPTOTIC VARIANCE OF ESTIMATORS

The asymptotic inconsistencies of maximum possibility estimators for the three parameters lognormal distribution can be obtained by overturning the Fisher material matrix, which are the negatives of the anticipated values of the subsequent partial spin-offs of the likelihood function through respect to the restrictions. Cohen et al. (1985) as in, Harter (1966) as in and Cohen (1980) as in found reasonable asymptotic variances through simulation studies. Based on the information matrix of $\hat{\lambda}$, $\hat{\mu}$ and ($\hat{\sigma}$) given by Hill (1963) as in, we have the asymptotic variances as

$$V(\lambda) = \frac{\sigma^2}{n}\left[\frac{(\exp(\mu))^2}{\exp(\sigma^2)}\right] \times [\exp(\sigma^2)(1+\sigma^2) - (1+2\sigma^2)]^{-1} \tag{13}$$

$$V(\mu) = \frac{\sigma^2}{n}\{1 + [\exp(\sigma^2)(1+\sigma^2) - (1+2\sigma^2)]^{-1}\} \tag{14}$$

$$V(\sigma) = \frac{\sigma^2}{2n}\{1 + 2\sigma^2[\exp(\sigma^2)(1+\sigma^2) - (1+2\sigma^2)]^{-1}\} \tag{15}$$

5. MONTE CARLO STUDY

A Monte Carlo education was showed to inspect the biasness of maximum likelihood estimates for$\hat{\lambda}$, $\hat{\mu}$ and ($\hat{\sigma}$) as well as the validity of the asymptotic and their applicability to samples of moderate size. The repetitive applications were attainedby the aid of processer programs using S-PLUS 2000 Professional.

A total of 50 distinct trials were complete. In alltrial of 2000 independent pseudo-random samples, a mixture of sample size n and the worth of the shape limit σ werecareful. In all cases the position and scale limitations (μ and λ) stood set at zero underprivileged of loss of oversimplification. The 50 experiments involved five sample sizes n= 25, 50, 100, 200 and 400 with ten values of the shape parameter σ = 0.2, 0.4, 0.6, 0.8, 1.0, 1.2, 1.4, 1.6, 1.8 and 2.0.

We provide detailed steps to investigate the biasness and the validity of maximum likelihood estimates of $\hat{\lambda}$, $\hat{\mu}$ and ($\hat{\sigma}$) for both method LMLE and MMLE as follows:

Step 1: Generate an independent three parameters lognormal distribution for variable X j of length n with location and scale parameters (μ and λ) are set to zero and shape parameter σ = 0.2, 0.4, 0.6, 0.8, 1.0, 1.2, 1.4, 1.6, 1.8 and 2.0.

Step 2: Compute ($\hat{\lambda}$) , $\hat{\mu}$ and ($\hat{\sigma}$) from equations (8) to (12), followed by the variances (λ)$\hat{V}$, V($\hat{\mu}$) and V($\hat{\sigma}$) of the parameters.

Step 3: Replication Step 1 besides Step 2 for m=2000 times to get the following:

Estimated bias of $B(\hat{\lambda}) = \bar{\hat{\lambda}} - 0$where $\bar{\hat{\lambda}} = \frac{1}{2000}\sum_{m=1}^{2000} \hat{\lambda}_m$since the true calculation of parameter λ is equal to 0

Estimated bias of $B(\hat{\mu}) = \bar{\hat{\mu}} - 0$ where $\bar{\hat{\mu}} = \frac{1}{2000}\sum_{m=1}^{2000} \hat{\mu}_m$ since the true value of parameter μ is equal to 0.

Estimated bias of $B(\hat{\sigma}) = \bar{\hat{\sigma}} - \sigma$,where $\bar{\hat{\sigma}} = \frac{1}{2000}\sum_{m=1}^{2000} \hat{\sigma}_m,$ since the true value of stricture μ is equal to 0.

Estimated bias of $B(\hat{\sigma}) = \bar{\hat{\sigma}} - \sigma,$ where $\bar{\hat{\sigma}} = \frac{1}{2000}\sum_{m=1}^{2000} \hat{\sigma}_m$ where the true value of stricture σ = 0.2, 0.4, 0.6, 0.8, 1.0, 1.2, 1.4, 1.6, 1.8 and 2.0.

Step 4: Compare each of the estimated variances with asymptotic variances as shown in Table 3.3

Table 1: Asymptotic Variance for Maximum Likelihood Estimators s (λμσ)

σ	n×V(λ)	n×V(μ)	n×V(σ)	n×V(β)	n×V(σ^2)
0.2	15.730	16.412	0.675	16.412	0.108
0.4	3.3.0	4.037	0.700	4.037	0.448
0.6	1.095	1.930	0.745	1.930	1.073
0.8	0.406	1.411	0.813	1.411	2.083
1	0.151	1.410	0.910	1.410	3.642
1.2	0.053	1.664	1.043	1.664	6.008
1.4	0.017	2.082	1.219	2.082	9.555
1.6	0.005	2.624	1.444	2.624	14.788
1.8	0.001	3.272	1.724	3.272	22.345
2	0.000	4.015	2.061	4.015	32.970

6. RESULTS AND DISCUSSION

Based on the results of the extensive sampling experiments as described previously, we now examine the biasness and the variances of the estimators as compared to the asymptotic modifications of the all-out likelihood estimators for both methods.

a) The Biasness The means of the estimates (λμσ) against σ for numerous values of n are given in Figure 1, 2, 3. As the true values for λ and μ are zero, these means are also their biases. As guideline, the asymptotic line represents as the true values of parameters.

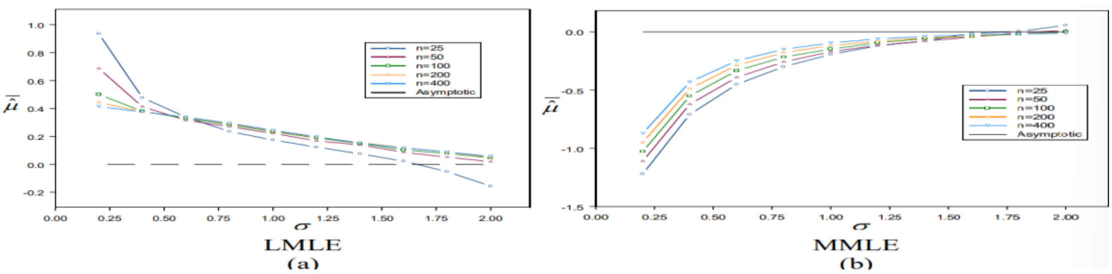

Figure 2: Plot of mean $\hat{\lambda}$against for σnumerous values of n

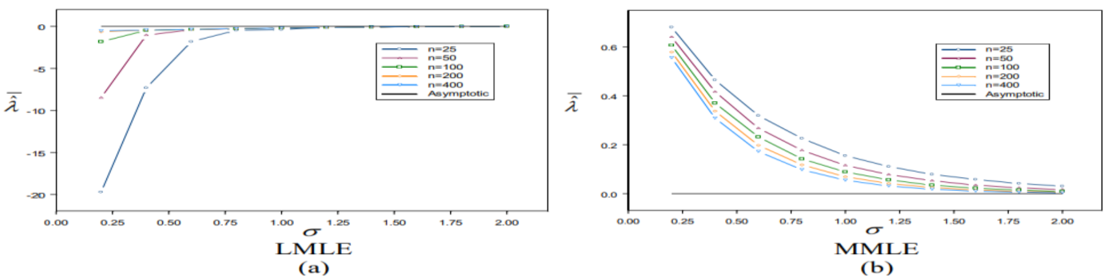

Figure 3: Plot of mean of $\hat{\mu}$ against σ for numerous values of n

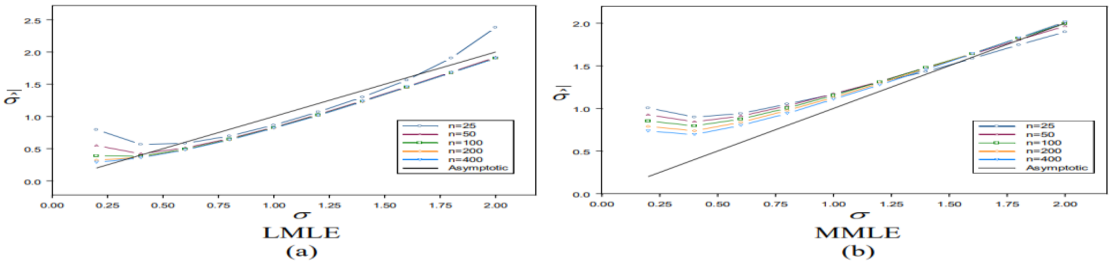

Figure 4: Plot of mean of $\hat{\sigma}$ against σ for numerous values of n

i) LMLE

Figure 2 (a) shows that the estimator $\hat{\lambda}$has a negative bias. The bias of $\hat{\lambda}$ is large and negative for small σ and decreases to almost zero as σ varies from 0.8 to 2.0. A similar pattern is seen in the bias for $\hat{\lambda}$ as n becomes large. On the other hand, Figure 3 (a) exhibits that the estimator $\hat{\mu}$ has positive bias. The bias becomes smaller as σ becomes larger. From the plot of $\hat{\mu}$against σ , the downward trend of the curves shows that there is a factor of $\exp(-\sigma)$ in the formulae for the mean of bias. Meanwhile, Figure 4 (a) visibly shows that the estimator $\hat{\sigma}$ has positive bias for small σ . There appears to be a linear trend in the graph of $\hat{\sigma}$ against σ and approaches the asymptotic line as σ becomes larger. A similar pattern in the bias for σ ˆ can be seen as n gets large.

ii) MMLE

The MMLE performance can be seen in Figure 2 (a), and 2 (b). The estimator $\hat{\lambda}$ has a positive bias as displayed in Figure 2(b). The bias becomes smaller as σ gets larger. A similar pattern is seen in $\hat{\lambda}$. From the plot of $\hat{\lambda}$ against σ, the downward trend of the curves confirms that there is a factor of $\exp(-\sigma)$ in the formulae for the mean of biases. In Figure 2 (b), the estimator $\hat{\mu}$ has negative bias. The bias of $\hat{\mu}$is large and negative for small σ and decreases to almost zero as σ varies from 1.6 to 2.0. The estimator σ ˆ has positive bias as demonstrated in Figure 3 (b). Similarly, there appears to be a linear trend in the graph of $\hat{\sigma}$ against σ and approach the asymptotic line has σ becomes larger. Again, the biases for σ can be understood as n converts very large.

b) The Asymptotic Variances The goodness of estimators for both methods are assessed further using their asymptotic variances as Table 1. Figure 5, 6, and 7 show the variances of the estimates ($\lambda\mu\sigma$) multiplied by n) against σ for innumerable values of n. In the plots, the black colour curves represent the asymptotic variances.

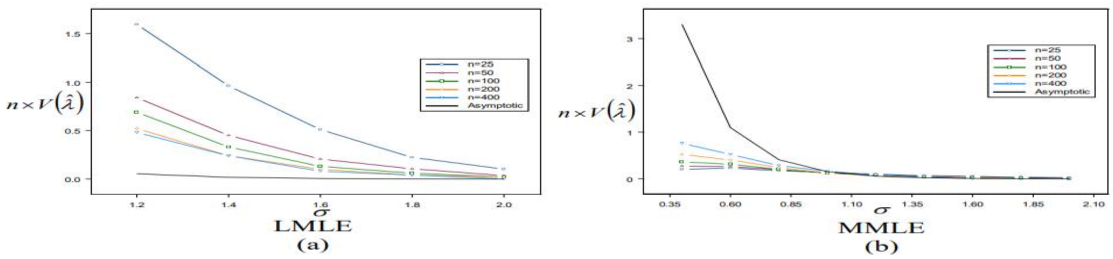

Figure 5: Plot of mean of nxV(λ) against σ for numerous values of n

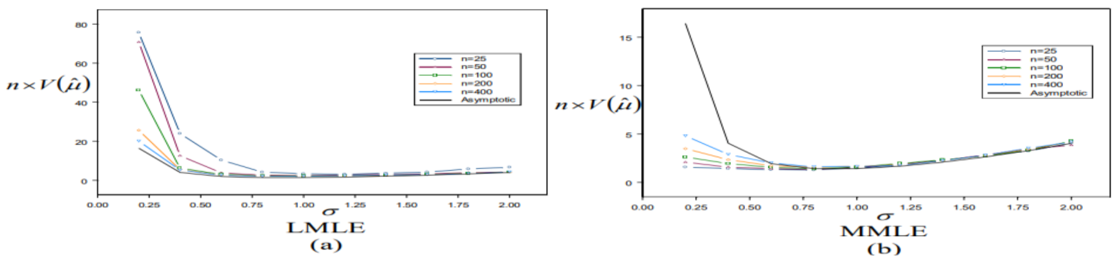

Figure 6: Plot of mean of n ×V($\hat{\mu}$) against σ for numerous values of n

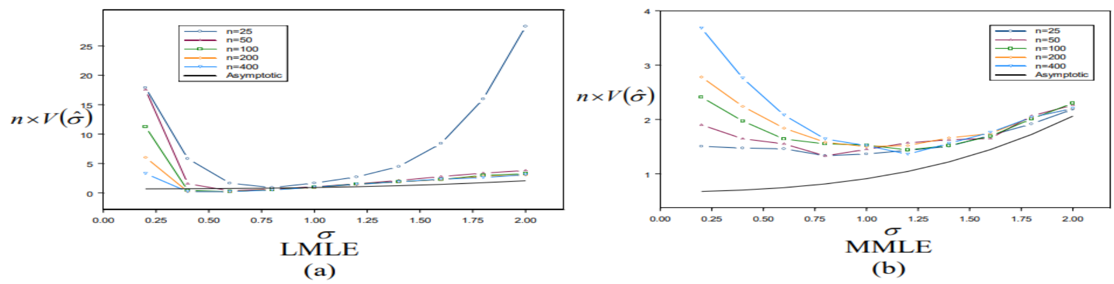

Figure 7: Plot of mean of n×V($\hat{\sigma}$) against σ for various values of n

i) LMLE

Figure 5 (a) illustrates the behaviour of)$\hat{n}\times V(\lambda)$ against σ which appears to follow the asymptotic curve as n and σ become larger. Meanwhile, in Figure 6 (a) and 7 (a), the variances of of$\hat{\mu}$ and $\hat{\sigma}$ appear to follow the asymptotic curve when n is large.

ii) MMLE

In Figure 5 (b), the variance of $\hat{\lambda}$ appears to follow the asymptotic curve when σ is near 1.0. As n becomes larger it tends to follow the asymptotic curve and converges to it.

The variance of $\hat{\mu}$appears to follow the asymptotic curve when σ is near 0.6 as in Figure 6 (b). Similarly, for the variance of $\hat{\lambda}$when n increases the variance of of$\hat{\mu}$ converges to the asymptotic curve. In Figure 7 (b), the variance of $\hat{\sigma}$ appears to follow the asymptotic curve when σ is near 1.0. However when σ becomes much smaller than 1.0 and n is large, the variation becomes larger. In general, we conclude that the asymptotic curve can be relied on for σ> 1.0. For the case of σ≤ 1.0, the asymptotic curve is unreliable as second order term becomes significant and cannot be ignored.

7. NUMERICAL EXAMPLE

In this section, we used the data of insurance claim cost for fatal and non-fatal accident from Insurance X (a well-known insurance company in Malaysia) to estimate parameters ($\lambda\mu\sigma^2$) by using both methods for LMLE and MMLE. We are considering the sample of 98 observations for fatal and 300 observations for non-fatal recorded from year 2006 to 2008.

Table 2: Estimates of ($\lambda\mu\sigma$) for accident claim cost from Insurance X.

Data	Method LMLE			Method MMLE		
	$\hat{\lambda}$	$\hat{\mu}$	$\hat{\sigma}$	$\hat{\lambda}$	$\hat{\mu}$	$\hat{\sigma}$
Fatal	1199	8.589	4.867	1200	8.588	4.867
Non-Fatal	1099	8.390	5.034	1100	8.391	5.034

From Table 2, it is noted that the estimates of λ,μ and σ either using LMLE or MMLE are not much differ from each other. This means that our proposed method can also be used to estimate the parameters for three parameters of lognormal delivery.

8. CONCLUSIONS:

In this study, we use a quasi-Newton-Raphson iteration in the 3 parameter lognormal spreading to propose a new MMLE. The effectiveness of the suggested technique was evaluated based on the attributes of its mean and variances, and these results were compared to the results obtained from the existing local maximum likelihood estimation (LMLE). A large number of simulation tests were carried out in order to investigate the biases and variances of the estimators in comparison to the asymptotic modification of the ML estimators. Our findings from the simulation experiments lead us to the conclusion that the method that was proposed provides a workable and accurate approximation of the parameters in the three parameters lognormal spreading.

The proposed method can handle two problems of likelihood function whereby the value of any ordered sample $x_1,x_2,\ldots,x_n$ will not tend to ∞ as ($\lambda\mu\sigma^2$) approach $x_1 - \infty\infty$.

This approach is also applicable to samples of any size, including those of a very minute proportion. In addition to this, the convergence issue that occurs in the Newton-Raphson iteration approach can also be circumvented. It has been demonstrated that the MMLE approach is effective in providing accurate estimates of the parameters for three-parameter lognormal distributions. As an example, this method has been applied to the data from insurance claims, and the findings are not significantly different from those obtained by the conventional method, which makes use of LMLE.

References:

- Stacy, E. W. (1962). A generalization of the gamma distribution. The Annals of Mathematical Statistics, 33, 11871192.
- Jaggia, S. (1991). Specification tests based on the heterogeneous generalized gamma model of duration: with an application to Kennan's strike data. Journal of Applied Econometrics, 6, 169-180.
- Kiefer, N. M. (1984). Simple test for heterogeneity in exponential models of duration. Journal of Labour Economics, 539 549.
- Lancaster, T. (1979). Econometric methods for the duration of unemployment. Econometrica, 939 -956.
- Sarmento, R. P. and Costa, V. (2019). An Overview of Statistical Data Analysis. Short studies, Research Gate, 1-29.
- Mishra, P., Pandey, C. M., Singh, U., Keshri, A., and Sabaretnam, M. (2019). Selection of appropriate statistical methods for data analysis. Annals of Cardiac Anaesthesia. 22(3), 297-301.
- Seufert, M. (2021). Statistical methods and models based on quality of experience distributions. Quality and User Experience, 6(3), 1-27.
- Savsani, M., and Ghosh, D. (2017). Bayesian inference for moderate distribution. Int. J. Agricult. Stat. Sci., 13(1), 303-317.
- Peligrad, M., Sang, H., Zhong, Y., andWu, W. B. (2018).Exact Moderate and Large Deviations for Linear Processes. *Math. St*, 1-28.

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CRANK-NICOLSON SCHEMEON BEHALF OF CRACKING SINGULARLY PERTURBED LINEAR PARABOLIC SYSTEMS

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Abstract: Using a Crank-Nicolson technique (CNT), we were able to solve the parabolic partial differential equations (PDE) in this study. The study examines two, analytical and numerical, approaches to solving PDE (finite difference method). The diffusion problem was numerically solved using the modified CNT, a finite difference approximation. The goal was to contrast the precise results of the modified CNT with the approximate results of a traditional method using the separation of variables. As the numerical method is simple and quick, solutions were acquired manually. The temperatures at particular time steps were compared to the corresponding analytical result. The outcomes were also tallied and shown.

Keywords: Crank-Nicolson technique, partial differential equations, diffusion problem

1. Introduction:

The general parabolic system of RD linear equations with a single perturbation is the focus of this paper. There is a lot of overlap between the different parts of the solution. A spatial delay parabolic SP DE has a perturbation parameter that doubles the highest order derivative and at least one delayed component on the spatial variable. Most commonly, one-dimensional heat equations with parabolic PDE are well-known equations (conduction equations). This kind of equation, which is said to be the most basic of all parabolic diffusion equations, can arise from problems where time is one of the independent variables. This kind of equation is crucial in a variety of real-world applications, including fluid mechanics. Analytical solutions to parabolic equations are difficult to find, and their applicability is further limited to problems requiring shapes for which boundary conditions are met.

Only a small number of parabolic equations can be solved in this method. Numerical techniques are among the few available methods of resolution in such circumstances. The literature provides many different uniform numerical techniques for resolving solitary perturbation problems.In this paper we measured the judgement of the adapted Crank-Nicolson structure to the exact explanation. There are numerical examples. The next section looks at the parabolic initial-boundary value issue.

$$\vec{L}\vec{u} = \frac{\partial \vec{u}}{\partial t} + \vec{L}_a\vec{u} = \vec{f} \text{ on } \Omega, \vec{u} \text{ given on } \Gamma, \qquad (1)$$

where the operator $\vec{L}_a$ is well-defined by

$$\vec{L}_a = -E\frac{\partial^2}{\partial a^2} + A$$

Here

$\Omega = \{(x,t): 0 < x < 1, 0 < t \le T\}$, $\bar{\Omega} = \Omega \cup \Gamma$, $\Gamma = \Gamma_L \cup \Gamma_B \cup \Gamma_R$ with $\vec{u}(0,t) = \vec{\phi}_L(t)$ on $\Gamma_L = \{(0,t): 0 \le t \le T\}$, $\vec{u}(x,0) = \vec{\phi}_B(x)$ on $\Gamma_B = \{(x,0): 0 < x < 1\}$, $\vec{u}(1,t) = \vec{\phi}_R(t)$ on $\Gamma_R = \{(1,t): 0 \le t \le T\}$. For all $(x,t) \in \bar{\Omega}$, $\vec{u}(x,t) = (u_1(x,t), u_2(x,t), \dots, u_n(x,t))^T$, $\vec{f}(x,t) = (f_1(x,t), f_2(x,t), \dots, f_n(x,t))^T$

$$E = \begin{pmatrix} \varepsilon_1 & 0 & \cdots & 0 \\ 0 & \varepsilon_2 & \cdots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & \varepsilon_n \end{pmatrix}, A(x,t) = \begin{pmatrix} a_{11}(x,t) & a_{12}(x,t) & \cdots & a_{1n}(x,t) \\ a_{21}(x,t) & a_{22}(x,t) & \cdots & a_{2n}(x,t) \\ \vdots & \vdots & & \vdots \\ a_{n1}(x,t) & a_{n2}(x,t) & \cdots & a_{nn}(x,t) \end{pmatrix}$$

The reduced problem associated with (1) is defined as

$$\frac{\partial \vec{u}_0}{\partial t} + A\vec{u}_0 = \vec{f} \text{ on } \Omega, \vec{u}_0 = \vec{u} \text{ on } \Gamma_B,$$

Assumption 1: The ε_1 are believed to be unique and, for convenience, ordered

$$\varepsilon_1 < \cdots < \varepsilon_n$$

Cases in which some of the parameters correspond are not taken into account.

Assumption 2: For all $(a,t) \in \bar{\Omega}$,The components are expected to bea$_{ij}(a,t)$of A (a,t)fulfill the inequalities

$$a_{ii}(x,t) > \sum_{\substack{j\neq i\\ j=1}}^{n} \left|a_{ij}(x,t)\right| \text{ for } 1 \le i \le n, \text{ and } a_{ij}(x,t) \le 0 \text{ for } i \ne j. \quad \textbf{(2)}$$

Assumption 3: There is a positive number that is equal to the inequality.

$$0 < \alpha < \min_{\substack{(x,t)\in\bar{\Omega}\\ 1\le i\le n}} \left(\sum_{j=1}^{n} a_{ij}(x,t)\right) \quad \textbf{(3)}$$

Assumption 4: without sacrificing generality, it is also believed that

$$\sqrt{\boldsymbol{\varepsilon}_{\mathbf{n}}} \le \frac{\sqrt{\mathbf{a}}}{\mathbf{6}} \qquad (\mathbf{4})$$

This ensures that all levels are included in the solution domain.

Assumption 5 Further $\vec{\mathrm{f}}$and Aare considered to be sufficiently smooth, with acceptable compatibility criteria, so that $\vec{u} \in C_{\lambda}^{(6)}(\bar{\Omega})$ for $A, \vec{f} \in C_{\lambda}^{(3)}(\bar{\Omega})$

$$C_{\lambda}^{k}(D) = \left\{u: \frac{\partial^{l+m}u}{\partial x^{l}\,\partial t^{m}} \in C_{\lambda}^{0}(D) \text{ for } l,m \ge 0, l \le 4, \text{ and } 0 \le l+2m \le k\right\}$$

2. ANALYTICAL RESULTS

The operator $\vec{\mathrm{L}}$complies with the maximal principle.

Assumption 6: Let A(a, t) mollify (2) and (3). Let $\vec{\psi}$be some vector-valued utility in the area of $\vec{\mathrm{L}}$s.t.$\vec{\psi} \ge \vec{0}$ on Γ. then $\vec{\mathrm{L}}\vec{\psi}(\mathrm{a},\mathrm{t}) \ge \vec{0}$on Ω implies that $\vec{\psi}(\mathrm{a},\mathrm{t}) \ge \vec{0}$on $\bar{\Omega}$

Assumption 7: Let A(a, t) gratify (2) and (3). If $\vec{\psi}$Where the domain of is any vector-valued function $\vec{\mathrm{L}}$, then, for each i, $1 \le i \le n$ and (a, t) $\in\bar{\Omega}$

$$|\psi_i(x,t)| \le max\left\{\parallel \vec{\psi} \parallel_{\Gamma}, \frac{1}{\alpha} \parallel \vec{L}\vec{\psi} \parallel\right\}$$

The Shishkindisintegration of the solution $\vec{\mathrm{u}}$ of (1)is $\vec{\mathrm{u}} = \vec{\mathrm{v}} + \vec{\mathrm{w}}$where the flat component $\vec{\mathrm{v}}$is the explanation of $\vec{\mathrm{L}}\vec{\mathrm{v}} = \vec{\mathrm{f}}$ in $\Omega, \vec{\mathrm{v}} = \vec{\mathrm{u}}_0$ on $\Gamma$and the singular constituent$\vec{\mathrm{w}}$is the answer of $\vec{\mathrm{L}}\vec{\mathrm{w}} = \vec{0}$ in $\Omega, \vec{\mathrm{w}} = \vec{\mathrm{u}} - \vec{\mathrm{v}}$ on Γ. For suitability the left and right limit layers of $\vec{\mathrm{w}}$additional decomposition is used to distinguish them $\vec{\mathrm{w}} = \vec{\mathrm{w}}^{\mathrm{L}} + \vec{\mathrm{w}}^{\mathrm{R}}$ where $\vec{\mathrm{L}}\vec{\mathrm{w}}^{\mathrm{L}} = \vec{0}$ on $\Omega, \vec{\mathrm{w}}^{\mathrm{L}} = \vec{\mathrm{w}}$ on $\Gamma_{\mathrm{L}}, \vec{\mathrm{w}}^{\mathrm{L}} = \vec{0}$ on $\Gamma_{\mathrm{B}} \cup \Gamma_{\mathrm{R}}$ and $\vec{\mathrm{L}}\vec{\mathrm{w}}^{\mathrm{R}} = \vec{0}$ on $\Omega, \vec{\mathrm{w}}^{\mathrm{R}} = \vec{\mathrm{w}}$ on $\Gamma_{\mathrm{R}}, \vec{\mathrm{w}}^{\mathrm{R}} = \vec{0}$ on $\Gamma_{\mathrm{L}} \cup \Gamma_{\mathrm{B}}$.

Defining layer functions $\mathrm{B}_{\mathrm{i}}^{\mathrm{L}}, \mathrm{B}_{\mathrm{i}}^{\mathrm{R}}, \mathrm{B}_{\mathrm{i}}, \mathrm{i} = 1, \dots, \mathrm{n},$, by

$$\mathrm{B}_{\mathrm{i}}^{\mathrm{L}}(\mathrm{a}) = \mathrm{e}^{-\mathrm{a}\sqrt{\mathrm{a}/\varepsilon_{\mathrm{i}}}}, \mathrm{B}_{\mathrm{i}}^{\mathrm{R}}(\mathrm{a}) = \mathrm{B}_{\mathrm{i}}^{\mathrm{L}}(1-\mathrm{a}), \mathrm{B}_{\mathrm{i}}(\mathrm{a}) = \mathrm{B}_{\mathrm{i}}^{\mathrm{L}}(\mathrm{a}) + \mathrm{B}_{\mathrm{i}}^{\mathrm{R}}(\mathrm{a})$$

Assumption 8: Let A(a, t) satisfy (2) and (3). Then the evenmodule$\vec{v}$of the answer$\vec{u}$ of (1)satisfies, for all $I = 1,\cdot\cdot\cdot, n$ and all $(a,t) \in \bar{\Omega}$

$$\left|\frac{\partial^l v_i}{\partial x^l}(x,t)\right| \le C\left(1+\sum_{q=i}^{n}\frac{B_q(x)}{\varepsilon_q^{\frac{1}{2}}-1}\right) \text{ for } l = 0,1,2,3$$

$$\left|\frac{\partial^l v_i}{\partial x^{l-1}\,\partial t}(x,t)\right| \le C \text{ for } l = 2,3.$$

Bounds on $\vec{w}^L, \vec{w}^R$as well as their results are found in

Assumption 9 Let A(a, t) fulfill (2) and (3). There is then a constant C s.t.aimed atboth$(a,t) \in \bar{\Omega}$ and $i = 1, \dots, n$.

$$\left|\frac{\partial' w_i^L}{\partial t^L}(x,t)\right| \le CB_n^L(x), \text{ for } l = 0,1,2, \left|\frac{\partial w_i^L}{\partial x^L}(x,t)\right| \le C\sum_{q=i}^{n}\frac{B_q^L(x)}{\varepsilon_q^{\frac{1}{2}}}, \text{ for } l = 1,2$$

$$\left|\frac{\partial^3 w_i^L}{\partial x^3}(x,t)\right| \le C\sum_{q=1}^{n}\frac{B_q^L(x)}{\varepsilon_q^{\frac{2}{2}}}, \left|\frac{\partial^t w_i^L}{\partial x^4}(x,t)\right| \le C\frac{1}{\varepsilon_i}\sum_{q=1}^{n}\frac{B_q^L(x)}{\varepsilon_q}.$$

The same holds true for the $w_i^R$and their results

3. CRANK-NICOLSON METHOD

On [0, T], a mesh by M mesh intermissions, as defined by $\bar{\Omega}_t^M = \{k\Delta t, 0 \le k \le M, \Delta t = T/M\}$is thought about On this mesh, the Crank-Nicolson technique is applied.

$$\begin{aligned}
&\vec{u}^0(x) = \vec{u}(x,0)\\
&\left(I+\tfrac{\Delta t}{2}\vec{L}_x\right)\vec{u}^{k+1}(x) = \tfrac{\Delta t}{2}\left(\vec{f}^k+\vec{f}^{k+1}\right)(x) + \left(I-\tfrac{\Delta t}{2}\vec{L}_x\right)\vec{u}^k(x) \quad \textbf{(5)}\\
&\vec{u}^{k+1}(0) = \vec{u}(0,t_{k+1}), \vec{u}^{k+1}(1) = \vec{u}(1,t_{k+1}), k = 0,\dots,M-1
\end{aligned}$$

Where $\vec{f}^k = \vec{f}(a,t_k), k = 0, \dots, M$

It is useful to pose the following fictitious problem:

$$\begin{aligned}
&\left(I+\tfrac{\Delta t}{2}\vec{L}_x\right)\vec{u}^{k+1}(x) = \tfrac{\Delta t}{2}\left(\vec{f}^k+\vec{f}^{k+1}\right)(x) + \left(I-\tfrac{\Delta t}{2}\vec{L}_x\right)\vec{u}(x,t_k), \quad (6)\\
&\vec{\hat{u}}(0) = \vec{u}(0,t_{k+1}), \vec{\hat{u}}^{k+1}(1) = \vec{u}(1,t_{k+1}),
\end{aligned}$$

where the answer$\vec{u}$ of (1) has substituted$\vec{u}^k$in the right-hand lateral of (5). the operator $I+\frac{\Delta t}{2}\vec{L}_a$complies with the maximal principle.

4. NUMERICAL ILLUSTRATIONS

This section focuses on two concerns. Using the suggested Crank-Nicolson technique, the results may be compared with the first order differential worker in t discretized by the Backward Euler scheme.

This problem may be addressed using multiple uniform meshes and a fine Shishkin mesh for ‘a’ in order to get an order of convergence in variable t. The two-mesh algorithm [FHM+00] is used to derive the parameter-uniform instruction of meeting and the error continuous.

The problem is then solved on a mesh $\overline{\Omega}^{M,N}$given a fixed M, suitably large, and various N values the a-order of convergence is then determined using the two-mesh approach.

The philosophy that Crank-Nicolson approach doubles the instruction of conjunctionbydeference to the adjustablet is adequately exemplified by both cases.

Example 1:

Consider the issue

$$\frac{\partial \vec{u}}{\partial t} - E\frac{\partial^2 \vec{u}}{\partial x^2} + A\vec{u} = \vec{f} \text{ on } (0,1) \times (0,1], \vec{u} = \vec{0} \text{ for } x = 0 \text{ or } t = 0 \text{ or } x = 1,$$

Where $E = \operatorname{diag}(\varepsilon_1, \varepsilon_2, \varepsilon_3), A = \begin{pmatrix} 6 & -1 & 0 \\ -t & 5(a+1) & -1 \\ -1 & -(1+a^2) & 6+a \end{pmatrix}, \vec{f} = \begin{pmatrix} 1+e^{a+t} \\ 1+a+t^2 \\ 1+e^t \end{pmatrix}$

To solve the aforementioned BVP, the Crank-Nicolson method is used. For different values of $\varepsilon_1, \varepsilon_2, \varepsilon_3$the maximum errors, ε-uniform order of conjunction and identical error constant are calculated. Consideration is given to the variation in all three parameters $\varepsilon_3 = \eta, \varepsilon_2 = \frac{\eta}{8}, \varepsilon_1 = \frac{\eta}{32}$where η as indicated in the tables.

Example 2:

Consider the issue

$$\frac{\partial \vec{u}}{\partial t} - E\frac{\partial^2 \vec{u}}{\partial a^2} + A\vec{u} = \vec{f} \text{on} (0,1) \times (0,1], \vec{u} = \vec{0} \text{ for } a = 0 \text{ or } t = 0 \text{ or } a = 1,$$

Where $E = \operatorname{diag}(\varepsilon_1, \varepsilon_2, \varepsilon_3), A = \begin{pmatrix} 4(1+a+t) & -t & -a \\ -2(1-t) & 7+\left((2+t)a\right) & -(3-a) \\ -1 & -(a+t) & 6\left(1+\frac{a}{2}+\frac{t}{2}\right) \end{pmatrix}$

$$\vec{f} = (1+a+t^2, 1+e^{a+t}, 2)^T$$

Table 1: Crank-Nicolson Method, t-convergence Example 1

η	Numberof mesh factsN				
	4	**8**	**16**	**32**	**64**
2^{-0}	0.665E-01	0.287E-01	0.133E-01	0.531E-02	0.186E-02
2^{-1}	0.633E-01	0.286E-01	0.112E-01	0.375E-02	0.182E-02
$S2^{-2}$	0.636E-01	0.253E-01	0.811E-02	0.371E-02	0.178E-02
2^{-3}	0.601E-01	0.198E-01	0.755E-02	0.360E-02	0.170E-02
2^{-4}	0.528E-01	0.161E-01	0.730E-02	0.345E-02	0.137E-02
2^{-5}	0.452E-01	0.150E-01	0.706E-02	0.281E-02	0.903E-03
D^N	0.665E-01	0.287E-01	0.133E-01	0.531E-02	0.186E-02
p^N	0.120E+01	0.111E+01	0.133E+01	0.150E+01	
C_P^N	0.573E+00	0.531E+00	0.531E+00	0.451E+00	0.34E+00
t-order of conjunction= 1.1					
The error continuous= 0.6					

Table 2: Crank-Nicolson Process, a-convergence Example 1

η	Number of lattice points N				
	24	**48**	**96**	**192**	**384**
2^{-0}	0.503E-02	0.131E-02	0.332E-03	0.836E-04	0.208E-04
2^{-1}	0.974E-02	0.258E-02	0.662E-03	0.166E-03	0.417E-04
2^{-2}	0.180E-01	0.502E-02	0.131E-02	0.331E-03	0.833E-04
2^{-3}	0.281E-01	0.972E-02	0.258E-02	0.661E-03	0. 166E-03
2^{-4}	0.356E-01	0.180E-01	0.501E-02	0.131E-02	0.331E-03
2^{-5}	0.357E-01	0.281E-01	0.972E-02	0.258E-02	0.661E-03
D^N	0.358E-01	0.281E-01	0.972E-02	0.258E-02	0.661E-03
p^N	0.345E+00	0.153E+01	0.191E+01	0.195E+01	
C_P^N	0.504E+00	0.504E+00	0.220E+00	0.747E-01	0.242E-01
a- order of union= 0.35					
The error continuous= 0.5					

Table 3: Crank-Nicolson Method, t-convergence Example 2

η	Number of mesh facts N				
	8	**16**	**32**	**64**	**128**
2^{-0}	0.311E-01	0.141E-01	0.654E-02	0.261E-02	0.851E-03
2^{-1}	0.292E-01	0.136E-01	0.540E-02	0.172E-02	0.683E-03
2^{-2}	0.294E-01	0.115E-01	0.371E-02	0.138E-02	0.443E-03

2^{-3}	0.264E-01	0.855E-02	0.285E-02	0.913E-03	0.228E-03
2^{-4}	0.214E-01	0.605E-02	0.193E-02	0.485E-03	0.206E-03
2^{-5}	0.211E-01	0.588E-02	0.182E-02	0.486E-03	0.215E-03
D^N	0.315E-01	0.141E-01	0.645E-02	0.261E-02	0.851E-03
p^N	0.115E+01	0.108E+01	0.132E+01	0.160E+01	
C_P^N	0.567E+00	0.541E+00	S0.541E+00	0.458E+00	0.322E+00

t-order of convergence= 1.1
The error continuous= 0.6

Conclusion

The dissimilarity in all three strictures is determined by taking into account $\varepsilon_3 = \eta, \varepsilon_2 = \frac{\eta}{4}, \varepsilon_1 = \frac{\eta}{16}$ where η as seen in the tables, there are a variety of options. 0.9 The problem is solved by setting N to 198 $\overline{\Omega}^{M,N}$by Crank-Nicolson scheme for different values of M and figuring out the order of meeting in the variable t. The problem is solved by making M equal to 32 $\overline{\Omega}^{M,N}$for dissimilar values of N, and the directive of convergence in the variable 'a' is figured out. Tables 1 and 3 show that the Modified CNT technique performs well, is consistent, and agrees with the exact answer when used to solve parabolic partial differential equations. Better than the traditional CNT, it offers quicker convergence and greater precision and necessitates tridiagonal system solutions at every level. Based on the foregoing discussion, it can be concluded that analytical approaches always yield precise solutions, while the modified CNT, in contrast to the classical CNT, yields approximations and exhibits rapid convergence. As not all PDEs can be solved analytically, numerical approaches offer good agreement when there are no solutions or when a PDE cannot be solved analytically. The outcomes of our approach also support earlier findings in the literature that smaller time steps result in more precise outcomes.

References:

- Wakjira, T. G., and Gemechis, F. D. (2021). Parameter-Uniform Numerical Scheme for Singularly Perturbed Delay Parabolic Reaction Diffusion Equations with Integral Boundary Condition. *International Journal of Differential Equations*, 1-16. https://doi.org/10.1155/2021/9993644.
- Salimova, D. (2019). Numerical approximation results for semi linear parabolic partial differential equations. 10.3929/ethz-b-000383156.
- E. O'Riordan, F. J. Lisbona, and J. L. Gracia. A coupled set of parabolic RDEs that have been individually perturbed. *Advances in Computer Mathematics*, 32(1):43-61, 2010.
- Elango, S., Tamilselvan, A., and Vadivel, R. *(2021).* Finite difference scheme for singularly perturbed reaction diffusion problem of partial delay differential equation with nonlocal boundary condition. *Adv Differ Equ* 2021, 151 (2021). https://doi.org/10.1186/s13662-021-03296-x.
- Kabeto, M.J., Duressa, G.F. (2021a). Accelerated nonstandard finite difference method for singularly perturbed Burger-Huxley equations. *BMC Res Notes* 14, 446. https://doi.org/10.1186/s13104-021-05858-4.
- S. Natesan and J. Mohapatra Second-order numerical technique for singly perturbed delay DEs that is uniformly convergent. 2008, *Neural Parallel Sci. Comput.*, 16(3):353-370.
- Kumar, M. Singh, J. Kumar, S. and Chauhan, A. (2021). A robust numerical method for a coupled system of singularly perturbed parabolic delay problems. *Engineering Computations*, 1-14.
- Thomas Y. H., Wang, Z., and Zhang, Z. (2019). A class of robust numerical methods for solving dynamical systems with multiple time scales. *Cornell University*, 1-24
- Burcu, G. (2021). A Computational Technique for Solving Singularly Perturbed Delay Partial Differential Equations. *Foundations of computing and decision sciences*, 46 (3), 12-26.
- Masho, J. and Gemechis, F. D. (2021b) -Robust numerical method for singularly perturbed semilinear parabolic differential difference equations. *Mathematics and Computers in Simulation*, 188, 537-547.

- Kabeto, M. J. and Duressa, G. F. (2021). Robust numerical method for singularly perturbed semilinear parabolic differensstial difference equations.*Mathematics and Computers in Simulation*, 537-547.
- Negero, N. T., and Duressa, G. F. (2021) Uniform Convergent Solution of Singularly Perturbed Parabolic Differential Equations with General Temporal-Lag. *Iran J SciTechnol Trans Sci*, 1-9. https://doi.org/10.1007/s40995-021-01258-2.
- Srivastava, S., and Banicescu, I. (2016). Robust resource allocations through performance modeling with stochastic process algebra. *Concurrency and Computation: Practice and Experience*, 1-12.

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The Sequence