Volume 37 June & December 2023 ISSN 0970-9169

GANITA SANDESH

गणित संदेश

A Half Yearly International Research Journal

of

Rajasthan Ganita Parishad

Registered Head Office

Department of Mathematics

SPC Government College, AJMER – 305001 (*INDIA*)

(NAAC Accredited "A" Grade College)

Website: www.rgp.co.in
E-mail: rgp@rgp.co.in

Issued September, 2024

E-mail: editor@rgp.co.in
E-mail: cmtr@rgp.co.in

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Ganita Sandesh Vol. 37 (June & December, 2023) pp. 1 - 5 ISSN : 0970-9169

A Note on Daubechies Polynomials

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Abstract: We exhibit the connection between the Daubechies polynomials, the Gauss hypergeometric function, and the modified Legendre polynomials.

Keywords: Shifted Legendrepolynomials, Gauss hypergeometric function, Daubechies polynomials.

1.- Introduction

The wavelets are very important in science, engineering and technology [1-4], in particular, the construction of Daubechies wavelets [5] depends strongly from the zeros of Daubechies polynomials$d_n(x)$ [6, 7], thus it is attractive to study the properties of these polynomials because their behavior gives useful information on the corresponding wavelets. Here we show that the analysis of$d_n(x)$maybe guided through the modified Legendre polynomials$P_l^*(x)$[8-18], thus a better understanding of Daubechies polynomials can be obtained via the Legendre polynomials.

The shifted Legendre polynomials [13], for $x \, \varepsilon \, [0,1]$:

$$P_0^* = 1, \quad P_1^* = 1 - 2x, \quad P_2^* = 1 - 6x + 6x^2, \quad P_3^* = 1 - 12x + 30x^2 - 20x^3,$$

$$P_4^* = 1 - 20x + 90x^2 - 140x^3 + 70x^4, \quad P_5^* = 1 - 30x + 210x^2 - 560x^3 + 630x^4 - 252x^5, \ldots \tag{1}$$

are solutions of the differential equation:

$$x(1-x)y'' - (2x-1)\, y' + l(l+1)\, y = 0, \tag{2}$$

and they can be generated via the expression:

$$P_l^*(x) = \sum_{k=0}^{l} (-1)^k \binom{l}{k} \binom{l+k}{k} x^k, \qquad l = 0, 1, 2, 3, \ldots \tag{3}$$

or in terms of the Gauss hypergeometric function [11, 12, 19, 20]:

$$P_l^*(x) = {}_2F_1(-l, l+1; 1; x). \qquad (4)$$

In Sec. 2 we indicate similarities between the shifted Legendre and Daubechies polynomials [6, 7].

2.- Daubechies polynomials

Ifin (2) we realize a simple change into the coefficient ofy':

$$x(1-x)\, y'' - (2x + l)\, y' + l(l+1)\, y = 0, \qquad (5)$$

then it is nice to discover that the Daubechies polynomials:

$$d_0 = 1, \qquad d_1 = 1 + 2x, \qquad d_2 = 1 + 3x + 6x^2, \qquad d_3 = 1 + 4x + 10x^2 + 20x^3, \qquad (6)$$

$$d_4 = 1 + 5x + 15x^2 + 35x^3 + 70x^4, \qquad d_5 = 1 + 6x + 21x^2 + 56x^3 + 126x^4 + 252x^5, \ldots$$

are solutions of (5).

Fig. 1 shows the polynomials (6), where only we can see their real roots; in general, their zeros are complex, for example, the roots of$d_6$are (0.1411+0.3421 i), (-0.1246+0.2832 i), (-0.2665+0.1073 i) and their conjugates. Besides, there we note that$d_l(0) = 1$ and$d_l(x) > 0$ if $x > 0$, thus the real zeros are negative.

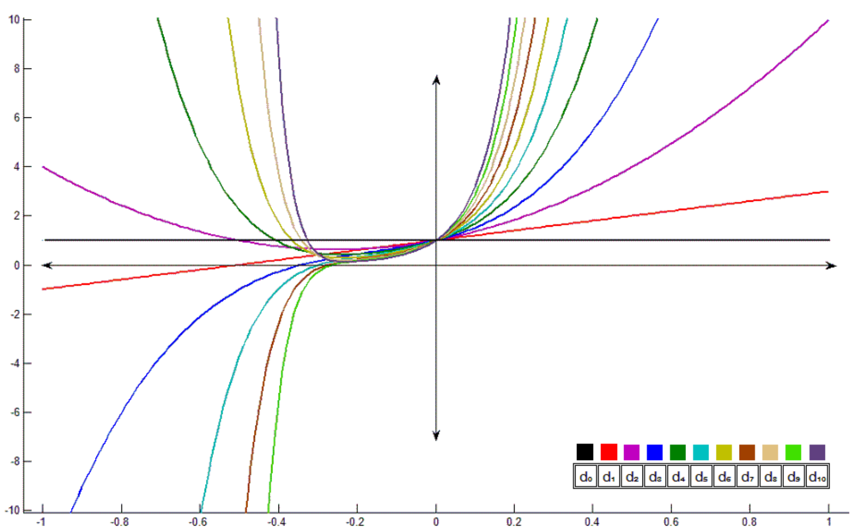

Figure 1.- Daubechiespolynomials.

The d_l are very important in the construction of the compactly supported Daubechies wavelets. There is a close relationship between the zeros of d_l and the $2l$ filter coefficients $h(l)$ of the Daubechies wavelets D_{2l} [6], therefore, it is fundamental to search efficient algorithms to find the roots of Daubechies polynomials, especially for large *l*. Here we show certain connections between (1) and (6), and then we hope that the stored experience with the roots of Legendre polynomials may be useful in the analysis of the zeros of (6).
It is easy to find the corresponding modification of (4):

$$d_l(x) = lim_{\lambda\to 0}\ {}_2F_1(-l, l+1; -l+\lambda; x)\,, \quad 0 < \lambda < 1, \tag{7}$$

hence (3) adopts the known form [6, 7]:

$$d_l(x) = \sum_{k=0}^{l} \binom{l+k}{l} x^k. \tag{8}$$

Thus, in the equation:

$$x(1-x)\, y'' - (2x+\alpha)\, y' + l(l+1)\, y = 0, \tag{9}$$

we have two cases of interest:

$$y(x) = \begin{cases} P_l^*(x), & \alpha = -1, \\ \\ d_l(x), & \alpha = l, \end{cases} \tag{10}$$

with the Rodrigues formulae:

$$P_l^*(x) = \frac{1}{l!}\frac{d^l}{dx^l}[x(1-x)]^l\,, \qquad d_l(x) = \frac{1}{l!}\left(\frac{x}{x-1}\right)^{l+1}\frac{d^l}{dx^l}\left[\frac{(x-1)^{2l+1}}{x}\right], \tag{11}$$

which generate to (1) and (6). The expression:

$$y(x) = \frac{1}{(l+1)!}[b + l + (1-b)(-1)^l]\left(\frac{x}{x-1}\right)^{b+l}\frac{d^l}{dx^l}\left[\frac{(x-1)^{b+2l}}{x^b}\right], \tag{12}$$

reproduces the relations (11) for $b = -l$ and $b = 1$, respectively.
From (8) we obtain the relations:

$$d_l(x) = \sum_{k=0}^{l} d_{lk}\, x^k\,, \qquad d_{lk} = \binom{l+k}{l}\,, \qquad k = 0, 1, \ldots, l \tag{13}$$

then it is immediate to deduce that:

$$d_{l0} = 1, \qquad d_{ll} = 2\, d_{l,l-1}\,, \qquad d_{lj} = \sum_{k=0}^{j} d_{l-1,k}\,, \quad j = 1, \ldots, l-1, \tag{14}$$

this means that the coefficients of $d_{l-1}(x)$ allow to construct the next $d_l(x)$; we note the following property of Daubechies polynomials:

$(1-x)^{l+1} d_l(x) + x^{l+1} d_l(1-x) = 1.$ (15)

The shifted Legendre polynomials verify the three-term recurrence relation [18, 21]:

$$(l+1)\, P^*_{l+1}(x) = (2l+1)(1-2x)\, P^*_l(x) - l\, P^*_{l-1}(x), \qquad (16)$$

which implies their orthogonality:
$\int_0^1 P^*_l(x)\, P^*_{l'}(x)\, dx = \frac{1}{2l+1}\delta_{ll'}$,(17)

however, the $d_l(x)$ are not orthogonal polynomials because they don't satisfy a three-term recurrence expression.

Remark 1.- The relation (7) can be written in the following simple form:

$$d_n(x) = \binom{2n}{n} x^n\ {}_2F_1\left(-n, 1;\ -2n;\ \frac{1}{x}\right). \qquad (18)$$

*Remark 2.-*We know the identity [22]:
$$\sum_{k=0}^{n} \binom{n+k}{n} \frac{1}{2^k} = 2^n\ , \qquad (19)$$

then (8) and (19) imply the property $d_n\left(\frac{1}{2}\right) = 2^n$, which can be verified with the polynomials (6).

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Ganita Sandesh Vol. 37 (June & December, 2023) pp. 6 - 12 ISSN : 0970-9169

OPTIMIZING OPERATIONS: THE USE OF INVENTORY CONTROL SYSTEMS AT SMART MARTS

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ABSTRACT

This article examines the incorporation of Inventory Control Systems (ICS) into the operations of SMART Marts, which are innovative retail stores that blend technology, convenience, and personalised shopping experiences. The paper explores the fundamental characteristics of ICS employed by SMART Marts and analyses the corresponding advantages, such as better precision, decreased costs, improved client experiences, and well-informed decision-making. The essay examines how the integration of existing literature and real-world experiences demonstrates the role of ICS in enhancing the operational efficiency and effectiveness of SMART Marts in today's retail industry.

Objective: This research seeks to examine the influence of Inventory Control Systems (ICS) on the operational efficiency of SMART Marts. Specifically, it focuses on how the incorporation of ICS boosts inventory management accuracy and improves the overall customer experience.

Keywords:

Inventory Control Systems, SMART Marts, Retail Operations, Real-time Monitoring, Automated Reordering, Barcode and RFID Technology, Data Analytics, Accuracy, Cost Reduction, Customer Experience, Decision-Making.

INTRODUCTION

In the ever-changing realm of retail, effective inventory management is essential for guaranteeing customer contentment, optimising revenues, and maintaining a competitive advantage. SMART Marts, like with other progressive firms, have used Inventory Control Systems (ICS) to optimise their operations.

Inventory control systems encompass many methodologies for managing and regulating inventory levels.

Inventory control systems utilise many strategies to efficiently manage and enhance the storage and transportation of items within a company. Various enterprises may employ a blend of these techniques depending on their particular requirements and the characteristics of their stock. **Below are many prevalent techniques for inventory management:**

1. ABC Analysis:

- This approach classifies inventory items into three categories (A, B, and C) according to their significance.
- Classifying items as A, B, or C is based on their value and priority. A items are characterised as high-value and high-priority, B items as moderate, and C items as low-value. This categorization aids in allocating resources and attention in a manner that is commensurate with their importance.

2. Just-In-Time (JIT):

- JIT is a strategy in which inventory is procured and delivered precisely when needed for manufacturing or sale, hence reducing the necessity of maintaining surplus stock.
- Necessitates meticulous collaboration with suppliers and a dependable supply chain to guarantee timely delivery of commodities.

3. Bulk Shipments:

- The process of procuring and receiving substantial amounts of inventory in a reduced number of shipments.
- Decreases the expenses associated with shipping and processing for each individual item, but necessitates a significant amount of storage space and entails the potential danger of maintaining surplus inventory.

4. **Safety goods:**

- The practice of holding a surplus amount of inventory to minimise the likelihood of running out of goods as a result of unexpected events such as sudden spikes in demand or disruptions in the supply chain.
- The determination of the safety stock level is influenced by factors such as the unpredictability of lead time and the uncertainty of demand.

5. The Economic Order Quantity

- EOQ is a method used to calculate the most efficient order quantity that minimises the expenses associated with keeping inventory and placing orders.

• Achieves a balance between the expenses incurred from maintaining surplus inventory and the expenses incurred from placing orders and replenishing stock.

6. FIFO (First-In, First-Out):

- FIFO is a method of inventory management that prioritises the use or sale of the oldest goods first, hence reducing the likelihood of obsolescence or spoilage.
- Particularly suitable for items with a short lifespan or susceptible to technological obsolescence.

7. LIFO (Last-In, First-Out):

- LIFO is a method of inventory management that operates on the assumption that the most recently acquired inventory is the first to be used or sold.
- While it can offer tax benefits, it may not accurately represent the physical movement of commodities.

8. Periodic Inventory System:

- Involves the laborious process of physically counting and documenting inventory at regular intervals, rather than continuously monitoring it.
- Ideal for smaller enterprises or those with less intricate inventory requirements.

9. Perpetual Inventory System:

- A system that keeps a continuously updated and accurate record of inventory levels in real-time, utilising advanced technology like barcode scanners and RFID.
- Offers precise and current information, but necessitates substantial initial configuration and continuous upkeep.

10. Forecasting of demand:

- Utilises previous sales data and market trends to forecast future product demand.
- Facilitates the optimisation of inventory levels by aligning them with projected demand, hence minimising the risk of stockouts or surplus inventory.

This paper examines the utilisation of Integrated Control Systems (ICS) by SMART Marts to optimise their inventory management procedures, boost precision, and ultimately provide customers with an improved shopping experience.

I. SMART Marts Overview:

SMART Marts are a modern retail concept that combines technology, convenience, and a customised purchasing experience (Smith et al., 2021). These establishments utilise state-of-the-art technologies to establish a smooth and effective shopping environment. Effective inventory management is a crucial component of SMART Marts, and the adoption of ICS (Inventory Control System) is essential for attaining operational excellence.

II. Main Characteristics of Inventory Control Systems:

Inventory Control Systems refer to a variety of technologies and procedures that are specifically developed to monitor, oversee, and enhance the movement of items within a retail setting (Chen et al., 2018). SMART Marts utilise the following essential characteristics:

1. Real-time Monitoring: The use of ICS allows SMART Marts to continuously monitor their inventory, reducing the likelihood of having excessive or insufficient stock levels (Li & Wang, 2019).
2. SMART Marts employ ICS to automate the process of reordering, hence minimising the chances of product shortages and optimising inventory levels (Doe, 2020).
3. Barcode and RFID Technology: By employing barcode and RFID technology, SMART Marts can efficiently and precisely monitor every item in their inventory, reducing mistakes and generating useful data for demand prediction (Jones & Smith, 2017).
4. Data Analytics: ICS enables SMART Marts to examine past sales data, consumer preferences, and seasonal patterns, assisting in making well-informed choices regarding inventory levels and advertising methods (Wu et al., 2022).

III. Advantages for SMART Marts:

1. Enhanced Precision: Through the reduction of manual data input and human errors, ICS guarantees the accuracy of inventory records, hence preventing any inconsistencies between the real stock levels and the recorded data (Gupta & Jain, 2018).
2. Cost Reduction: Implementing an efficient inventory management system, such as ICS, enables SMART Marts to prevent excessive stock, hence decreasing storage expenses and minimising the necessity for markdowns on surplus goods (Tan et al., 2019). In addition, automated operations result in time and labour cost savings.

3. Improved client Experience: By having precise inventory information, SMART Marts can efficiently meet client demands, reducing instances of out-of-stock items and guaranteeing a satisfactory shopping experience (Chen & Lee, 2016). As a result, client happiness and loyalty are heightened.
4. Strategic Decision-Making: The data and insights supplied by ICS enable SMART Mart managers to make strategic decisions, such as optimising product assortments and recognising high-demand items (Li & Wang, 2019).

Conclusion

In summary, the incorporation of Inventory Control Systems (ICS) in SMART Marts signifies a crucial progression in contemporary retail operations. An examination of the main characteristics and advantages linked to the adoption of ICS demonstrates a significant and positive effect on the operational effectiveness of SMART Marts. The combination of academic research and practical illustrations highlights the importance of ICS (Inventory Control System) in attaining precise inventory management, minimising expenses, and providing an enhanced shopping experience.

SMART Marts are at the forefront of technological innovation in the retail sector due to their use of Real-time Monitoring, Automated Reordering, Barcode and RFID Technology, and Data Analytics. These features not only improve the precision of inventory records but also enable decision-makers with useful insights, resulting in strategic and well-informed decisions.

Recommendations for Subsequent Investigations:

1. Investigate the possibility of incorporating AI algorithms into ICS to improve the accuracy of demand forecasts and automate decision-making.
2. Customer Engagement and Personalisation: Explore the potential of ICS to enhance the shopping experience by customising inventory according to specific customer preferences and behaviours.
3. This study investigates the impact of Inventory Control Systems (ICS) on sustainable practices, namely by minimising surplus inventory, decreasing waste, and optimising the sustainability of the supply chain.
4. Evaluate the viability and advantages of integrating blockchain technology into inventory control systems (ICS) to improve security, transparency, and traceability of inventory movements.

5. Perform a comparative analysis between SMART Marts utilising ICS and traditional retail models to measure the operational enhancements and cost effectiveness.

To recap, conducting additional study on cutting-edge technology, inventive approaches, and environmentally-friendly methods in the field of Inventory Control Systems will enhance the continuous prosperity and competitiveness of contemporary retail establishments as SMART Marts continue to develop.

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Ganita Sandesh Vol. 37 (June & December, 2023) pp. 13 - 18 ISSN : 0970-9169

"Quantitative Methods in Economic Analysis:

Harnessing the Power of Mathematics"

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Abstract

Quantitative methods play a pivotal role in economic analysis by providing rigorous frameworks for understanding complex economic phenomena. This paper explores the use of mathematics in economic analysis and highlights its power in enhancing our understanding of economic systems. By harnessing mathematical tools such as optimization techniques, differential equations, game theory, stochastic processes, and network theory, economists are able to develop sophisticated models that capture the dynamics of economic behavior and market interactions. Through the lens of mathematical finance, time series analysis, and economic forecasting, this paper illustrates how quantitative methods enable economists to uncover patterns in economic data and make informed predictions about future trends. By leveraging the power of mathematics, economists can gain deeper insights into economic dynamics and inform policy decisions for promoting sustainable economic growth.

Keywords : Quantitative methods , Economic analysis, Mathematical modelling, Economic forecasting.

Introduction

Economics, as a social science, endeavors to comprehend and analyze the intricate dynamics of human behavior, resource allocation, and market mechanisms. While economics primarily deals with qualitative concepts, the integration of mathematics has revolutionized the discipline, facilitating precise analysis, rigorous modeling, and insightful predictions. This paper elucidates the pervasive influence of mathematics in economics, spanning from foundational principles to cutting-edge research.

This research delves into the indispensable role of mathematics in economics, elucidating its multifaceted applications and pivotal contributions to economic theory, modeling, and policy formulation. By examining the historical evolution, contemporary methodologies, and future prospects, this article underscores the symbiotic relationship between mathematics and economics, accentuating the interdisciplinary synergy that propels advancements in both fields.

Historical Evolution

The fusion of mathematics and economics traces back to the pioneering works of scholars like Leon Walras, Vilfredo Pareto, and Alfred Marshall in the late 19th century. Their endeavors to formalize economic theories laid the groundwork for mathematical economics, culminating in the emergence of neoclassical economics. The subsequent advent of game theory, optimization techniques, and econometrics further cemented the integration of mathematics into economic analysis, fostering a rich tapestry of methodologies.

Leon Walras is renowned for his significant contribution to economics through the development of the modern theory of general economic equilibrium. His work, particularly highlighted in his book "Élémentsd'économie politique pure," has been instrumental in mathematizing economics and establishing the foundation for general equilibrium theory

Vilfredo Pareto's contributions to economics include the development of Pareto's Principle, Pareto's Law of Income Distribution, and his analysis of economic inequality. He also introduced structural-functional analysis and contributed to the field through his work "Cours d'économie politique."

Alfred Marshall's contributions to economics include the development of welfare economics, price elasticity, the theory of demand and supply, the theory of production, neoclassical economics, and the economics of industry.

Quantitative Methods in Economic Analysis

Quantitative methods in economic analysis involve the use of mathematical models and statistical techniques to analyze economic data. These methods are used to test economic theories, estimate economic relationships, and make predictions about future economic events. Some of the most common quantitative methods used in economics include:

1. **Regression analysis**: This is a statistical technique used to estimate the relationship between two or more variables. Regression analysis is used to estimate the coefficients of a regression equation, which can then be used to make predictions about the dependent variable given the values of the independent variables.

2. **Time-series analysis**: This method is used to analyze data that is collected over time, such as stock prices, interest rates, or GDP. Time-series analysis involves the use of statistical techniques to identify patterns and trends in the data, and to make predictions about future values.

3. **Simulation**: This method involves the use of mathematical models to simulate economic phenomena. Simulation can be used to test the robustness of economic theories, to explore the impact of different policy interventions, or to make predictions about future economic events.

4. **Optimization**: This method involves the use of mathematical optimization techniques to find the best solution to a particular problem. Optimization is used in economics to find the optimal allocation of resources, to maximize profits, or to minimize costs.

5. **Interpolation and Extrapolation** :Interpolation and extrapolation are statistical methods of estimation and forecasting. Interpolation is a statistical technique, which through a study of the time series of known figures of population allows us to make a data insertion between a given data set. On the other hand, extrapolation allows us to forecast or anticipate a value for some future date.

Applications of Quantitative Methods in Economic Analysis

Quantitative methods have been applied in a wide range of economic fields, including:

1. **Macroeconomics**: Quantitative methods are used to estimate the impact of monetary and fiscal policy on economic growth, inflation, and unemployment.
2. **Microeconomics**: Quantitative methods are used to estimate the impact of changes in prices, wages, or other economic variables on consumer behavior and firm behavior.
3. **Finance**: Quantitative methods are used to estimate the risk and return of financial assets, to price financial derivatives, and to make predictions about future financial market movements.
4. **International economics**: Quantitative methods are used to estimate the impact of trade policies on economic growth, to analyze the effects of exchange rate fluctuations on international trade, and to make predictions about future international economic events.
5. **Public economics**: Quantitative methods are used to estimate the impact of government policies on economic welfare, to analyze the effects of taxation on economic behavior, and to make predictions about future public sector outcomes.
6. **Environmental Economics** : Since its development in the late 1980s, ecological economics has been referred to as the multidisciplinary and transdisciplinary science of sustainability. Since then, it has combined basic and practical research in an effort to inform and transform society, governance, and environmental policy.The field's pioneer, Herman Daly, built on Nicholas Georgescu-Roegen's entropy economics by emphasizing a sustainable, quantifiable economic scale and attaining fairness in the allocation and management of economic gains. In order to achieve a just distribution, he advocated for both discursive tactics and quantitative assessments of economic size. (Haddad & Solomon, 2024)

Impact of Quantitative Methods on Economic Analysis

The use of quantitative methods in economic analysis has had a significant impact on the field of economics. Some of the key impacts include:

1. **Increased precision**: Quantitative methods have allowed economists to make more precise predictions about economic phenomena, and to test economic theories with greater rigor.
2. **Increased rigor**: Quantitative methods have led to greater rigor in economic analysis, as economists are required to provide more detailed and precise explanations of their methods and findings.
3. **Greater collaboration**: Quantitative methods have facilitated greater collaboration between economists and other social scientists, as well as between economists and practitioners in other fields such as finance and public policy.
4. **Increased relevance**: Quantitative methods have made economic analysis more relevant to policymakers and practitioners, as they provide a more empirical basis for making decisions and predictions.
5. **Economic Research** :The use of quantitative techniques has significantly influenced economic research by enabling rigorous analysis and empirical testing of economic hypotheses. Through the application of statistical models, regression analysis, and mathematical optimization, economists can make informed decisions based on data-driven evidence. The Journal of Quantitative Economics stands as a testament to the importance of quantitative research in advancing econometrics and mathematical economics within developing economies.

Conclusion

The symbiotic relationship between mathematics and economics epitomizes the interdisciplinary synergy that drives intellectual progress and societal advancement. From theoretical abstractions to empirical realities, mathematics empowers economists to unravel the complexities of economic phenomena, inform policy decisions, and foster inclusive prosperity. As we navigate the complexities of a rapidly evolving global economy, the integration of mathematics will continue to be indispensable in shaping our understanding of economic dynamics and crafting informed policy responses.

The integration of quantitative techniques in economics has revolutionized the field by providing researchers with powerful tools to analyze economic phenomena objectively. The applications of these methods are vast and varied, impacting areas such as financial economics, econometrics, and business research. Moving forward, further exploration and development of quantitative techniques will continue to shape the landscape of economic analysis and decision-making.

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Ganita Sandesh Vol. 37 (June & December, 2023) pp. 19 - 28 ISSN : 0970-9169

Five Dimensional LRS Bianchi type-V String Cosmological model with Dark Energy in Saez-Ballester Theory

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Abstract. In this paper, we have investigated five dimensional LRS Bianchi type –V string cosmological model with dark energy in scalar tensor theory of gravitation proposed by Saez-Ballester [1]. The exact solution of the field equations is established by assuming the condition of Berman law's and the shear scalar is proportional to scalar expansion. Some physical and geometrical properties are discussed comparing with the current observational findings.

Keywords: LRS Bianchi type -V, cosmic strings, dark energy, Saez-Ballester theory.

Introduction: The coupling of gravitational forces with other forces in substance is not feasible in the usual four dimensional space times. Therefore, higher dimensional theory as it may be useful at very early stages of the evolution of the universe. Actually, as time elaborate, the standard dimensions expand while the more dimensions' shrink to the Planckian dimension,which is far away our ability to find out with the recently available experimental facilities Chatterjee et al. [2]. Many authors have attracted to describe the problems in the field of higher dimensions Appelquist et al. [3], Chodos and Detweller [4] investigated massive string cosmological model in higher dimensional homogeneous space time in general relativity. In current years there has been a much interest in cosmic strings and string cosmological models. The concept of string theory was establishing to express events the early stage of the universe Kibble [5] and Vilenkin [6]. Many researchers (Hogan and Rees [7], Myung et al. [8], Krori et al. [9], Banerjee et al. [10], Gundalach and Ortiz [11], Barros and Romero [12], Yavuz and Yilmaz [13], Sen et al. [14], Sen [15], Bhattacharjee and Baruah [16], Barros et al. [17], Rahaman et al. [18], Reddy [19] established string cosmological models in various theories of gravitation. Recently, there has been tremendous interest in cosmological models with dark energy in Einstein's theory of general relativity because of our observable universe is accelerated expansion. It has been studied by many researchers such as Riess et al. [20], Perlmutter et al. [21], same authors like Caldwell [22] and Haunge [23] have studied cosmic microwave background (CMB) anisotropy and Daniel et al. [24] have discussed the large scale structure and surely suggest that dark energy influence the present universe through cosmic acceleration. According to these observations, researchers have accepted that dark energy is a fluid with negative pressure, making up around 70% of the present universe energy content to be behind for this acceleration due to repulsive gravitation. Researchers have studied many candidates for dark energy to fit the recent observations like cosmological constant, Tachyon, quintessence, phantom etc. Evolution of the equation of state (EoS) od dark energy $w_{de} = p_{de}/\rho_{de}$ carry from $w_{de} > -1$ in the near past (quintessence region) to$w_{de} < -1$at current stage (phantom region). The concept of dark energy was presented for understanding this recently accelerating expansion of the universe.

Saez-Ballester equations for the combined scalar and tensor fields are given by

$$R_{ij} - \frac{1}{2}Rg_{ij} - \omega\phi^n\left(\phi_{,i}\phi_{,j} - \frac{1}{2}g_{ij}\phi_{,k}\phi^{,k}\right) = -T_{ij} \tag{1}$$

And the scalar field satisfies the equation

$$2\phi^n\phi^{;i}_{;j} + n\phi^{n-1}\phi_{,k}\phi^{,k} = 0 \tag{2}$$

Where R_{ij} is the Ricci tensor, R is the scalar curvature, T_{ij} represents the energy-momentum tensor, ω is a dimensionless coupling constant.

The energy conservation equation is expressed by

$$T^{ij}_{;j} = 0 \tag{3}$$

Recently, few dark energy models with constant DP have been studied by Kumar and Singh [25], Akarsu and Kilinc [26-27], Yadav et al. [28], A.K Yadav and L-Yadav [29], Pradhan et al. [30] and Rao et al. [31-32]. Five dimensional dark energy models have been investigated by Reddy et al. [33] in a Saez-Ballester [1] scalar tensor theory of gravity. Saez and Ballester [1] generated a scalar-tensor theory of gravitation where the metric is coupled with a dimensionless scalar field in a simple manner. This coupling presents an adequate description of weak fields. The Saez-Ballester theory propose a plausible way to solve the Friedmann-Robertson-Walker (FRW) cosmologies. In this theory, two models have been broadly studied. First is Friedmann-Robertson-Walker and second is spatially homogeneous anisotropic models. In this theory, Singh and Agrawal [34-35], Ram and Singh [36], Ram and Tiwari [37], Singh and Ram [38], Reddy and Rao [39], Reddy [40], Mohanty and Sahu [41], Reddy et al. [42], and Rao et al. [43-44] are few of the researchers who have studied the cosmological models.

Investigated from the above research work, in this paper we will examined LRS Bianchi type -V universe filled with string and dark energy in Saez-Ballester Theory. We have structured this paper as follows. Section II contains the field equation for Bianchi type –V universe. Section IV represents the solutions for field equations. Physical parameters are evaluated in section III. Section V contains conclusion for this paper.

II Metric and field equations

We consider a five-dimensional LRS Bianchi type-V metric

$$ds^2 = dt^2 - A^2dx^2 - B^2e^{2x}(dy^2 + dz^2) - C^2dm^2 \tag{4}$$

Where, A, B, and C are functions of cosmic time t only. Now, the energy momentum tensor for two non-interacting fluids is given by,

$$T_{ij} = T_{ij}^{CS} + T_{ij}^{DE} \tag{5}$$

Here T_{ij}^{CS} and T_{ij}^{DE} are energy momentum tensors of one dimensional cosmic string and dark energy. Now the energy momentum tensor for cosmic strings is given by

$$T_{ij}^{CS} = (\rho + p)u_iu_j - pg_{ij} + \lambda x_ix_j \tag{6}$$

Where, $u^iu_i = -x^ix_i = 1$ and $u^ix_i = 0$. Here u^i denotes the four velocity vector and x^i is the direction of cosmic strings (along x-direction). p denotes pressure of the fluid, λ is the tension density of the strings and ρ represent the rest energy density of strings. Here λ may be positive and negative.

The energy momentum tensor for the Dark Energy is given by

$$T_i^{j(DE)} = (\rho + p)u_iu_j - pg_{ij}, \tag{7}$$

$$T_i^{j(DE)} = diag[-\omega_x, -\omega_y, -\omega_z, 1, -\omega_\Psi]\rho_{de}, \tag{8}$$

$$T_i^{j(DE)} = diag[-\omega_{de}, -(\omega_{de} + \beta), -(\omega_{de} + \gamma), 1, -(\omega_{de} + \gamma)]\rho_{de}, \tag{9}$$

Where, ω_{de} denotes the equation of state parameter (EoS) for Dark Energy, Dark Energy density is represents by ρ_{de}, β and γ are two skewness parameters and both are represents the deviations from ω_{de} along the y, z and *Ψ* axes respectively. To obtain the solution, we have taken $\omega_{de_x} = \omega_{de}$ and $\omega_{de_z} = \omega_{de_\Psi} = \omega_{de} + \gamma$. The Saez-Ballester field equations (1) and (2) for the metric (4) as a result (5), (6), (7), (8) and (9) can be written as:

$$2\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + 2\frac{\dot{B}}{B}\frac{\dot{C}}{C} + \frac{\dot{B}^2}{B^2} - \frac{1}{A^2} - \frac{1}{2}\omega\phi^n\dot{\phi}^2 = -\big((p + \lambda) + \omega_{de}\rho_{de}\big) \tag{10}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{A}}{A}\frac{\dot{B}}{B} + \frac{\dot{B}}{B}\frac{\dot{C}}{C} + \frac{\dot{A}}{A}\frac{\dot{C}}{C} - \frac{1}{A^2} - \frac{1}{2}\omega\phi^n\dot{\phi}^2 = -(P + (\omega_{de} + \beta)\rho_{de}) \tag{11}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{A}}{A}\frac{\dot{B}}{B} + \frac{\dot{B}}{B}\frac{\dot{C}}{C} + \frac{\dot{A}}{A}\frac{\dot{C}}{C} - \frac{1}{A^2} - \frac{1}{2}\omega\phi^n\dot{\phi}^2 = -(P + (\omega_{de} + \gamma)\rho_{de}) \tag{12}$$

$$2\frac{\dot{A}}{A}\frac{\dot{B}}{B}+2\frac{\dot{B}}{B}\frac{\dot{C}}{C}+\frac{\dot{A}}{A}\frac{\dot{C}}{C}+\frac{\dot{B}^2}{B^2}-\frac{3}{A^2}+\frac{1}{2}\omega\phi^n\dot{\phi}^2=\rho+\rho_{de} \quad (13)$$

$$\frac{\ddot{A}}{A}+2\frac{\ddot{B}}{B}+2\frac{\dot{A}}{A}\frac{\dot{B}}{B}+\frac{\dot{B}^2}{B^2}-\frac{3}{A^2}+\frac{1}{2}\omega\phi^n\dot{\phi}^2=-(P+(\omega_{de}+\gamma)\rho_{de}) \quad (14)$$

$$\frac{\dot{A}}{A}-\frac{\dot{B}}{B}=0 \quad (15)$$

$$\ddot{\phi}+\dot{\phi}\left(\frac{\dot{A}}{A}+2\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right)+\frac{n}{2}\frac{\dot{\phi}^2}{\phi}=0 \quad (16)$$

Where overhead dot denotes ordinary differentiation with respect to cosmic time t.

III Physical parameters:

The spatial volume and average scale factor is

$$V=a^4(t)=AB^2C \quad (17)$$

Mean Hubble parameter H:

$$H=\frac{\dot{a}}{a}=\frac{1}{4}\left(\frac{\dot{A}}{A}+2\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right) \quad (18)$$

Scalar expansion θ :

$$\theta=4H=\left(\frac{\dot{A}}{A}+2\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right) \quad (19)$$

Shear scalarσ^2 :

$$\sigma^2=\frac{1}{2}\left(\sum_{i=1}^{4}H_i^2-4H^2\right) \quad (20)$$

Mean anisotropy parameterA_h is given by

$$A_h=\frac{1}{4}\sum_{i=1}^{4}\left(\frac{H_i-H}{H}\right)^2 \quad (21)$$

Where H_i represents the directional Hubble parameter along x, y, z and m directions.

IV Solution of field equations

By integrating the equation (15), we obtain

$$B=kA \quad (22)$$

Where k is the constant of integration. Without loss of generality if we put $k=1$, then we get

$$B=A \quad (23)$$

from the equation (23), the field equations (10)- (16) and eq. of motion (3) will be

$$2\frac{\ddot{A}}{A}+\frac{\ddot{C}}{C}+2\frac{\dot{A}}{A}\frac{\dot{C}}{C}+\frac{\dot{A}^2}{A^2}-\frac{1}{A^2}-\frac{1}{2}\omega\phi^n\dot{\phi}^2=-\big((p+\lambda)+\omega_{de}\rho_{de}\big) \quad (24)$$

$$2\frac{\ddot{A}}{A}+\frac{\ddot{C}}{C}+\frac{\dot{A}^2}{A^2}+2\frac{\dot{A}}{A}\frac{\dot{C}}{C}-\frac{1}{A^2}-\frac{1}{2}\omega\phi^n\dot{\phi}^2=-(P+(\omega_{de}+\beta)\rho_{de}) \quad (25)$$

$$2\frac{\ddot{A}}{A}+\frac{\ddot{C}}{C}+\frac{\dot{A}^2}{A^2}+2\frac{\dot{A}}{A}\frac{\dot{C}}{C}-\frac{1}{A^2}-\frac{1}{2}\omega\phi^n\dot{\phi}^2=-(P+(\omega_{de}+\gamma)\rho_{de}) \quad (26)$$

$$3\frac{\dot{A}}{A}\frac{\dot{C}}{C}+3\frac{\dot{A}^2}{A^2}-\frac{3}{A^2}+\frac{1}{2}\omega\phi^n\dot{\phi}^2=\rho+\rho_{de} \quad (27)$$

$$\frac{\ddot{A}}{A}+2\frac{\ddot{B}}{B}+2\frac{\dot{A}}{A}\frac{\dot{B}}{B}+\frac{\dot{B}^2}{B^2}-\frac{3}{A^2}+\frac{1}{2}\omega\phi^n\dot{\phi}^2=-(P+(\omega_{de}+\gamma)\rho_{de}) \tag{28}$$

$$\ddot{\phi}+\dot{\phi}\left(3\frac{\dot{A}}{A}+\frac{\dot{C}}{C}\right)+\frac{n}{2}\frac{\dot{\phi}^2}{\phi}=0 \tag{29}$$

$$\dot{\rho}+4H(p+\rho)+\lambda H_1+\dot{\rho}_{de}+4H(\omega_{de}+1)\rho_{de}+\big(\beta H_2+\gamma(H_3+H_4)\big)\rho_{de}=0 \tag{30}$$

The acceptance of non-interacting behaviour of cosmic strings and Dark energy leads to these two individual equations from equation (30),

$$\dot{\rho}+4H(p+\rho)+\lambda H_1=0, \tag{31}$$

$$\dot{\rho}_{de}+4H(\omega_{de}+1)\rho_{de}+(\beta H_2+\gamma(H_3+H_4))\rho_{de}=0. \tag{32}$$

Eqs (24) - (29) are a system of six independent equations of following unknowns $A, C, D, \phi, \rho, \rho_{de}, \omega_{de}, p, \lambda, \beta$ *and* γ. Furthermore, the above six equations are highly non-linear equations. So, we take following condition to get a specified solution.

(i) The shear scalar σ^2 is proportional to scalar expansion θ so we can take (Collins et al. [45])

$$C=A^n \tag{33}$$

Where $n\neq 0$ is a constant.

(ii) By using the special law of variation for Hubble's parameter given by Berman [46] that gives constant deceleration parameter of the universe given by

$$q=-\frac{a\ddot{a}}{\dot{a}^2}=\text{constant} \tag{34}$$

By eq. (17) get the solution

$$V(t)=(AB^2C)^{\frac{1}{4}}=\left(A^{\frac{n+3}{4}}\right)=(ct+d)^{\frac{1}{(q+1)}} \tag{35}$$

Where c and d are constants of integration. Now from the eq. (23), (33) and (35), we obtain the solutions for the metric potentials as

$$A=B=(ct+d)^{\frac{4}{(n+3)(q+1)}},\ C=(ct+d)^{\frac{4n}{(q+1)(n+3)}} \tag{36}$$

By using the eq. (36) we obtain the metric (4) as

$$ds^2=dt^2-(ct+d)^{\frac{8}{(n+3)(q+1)}}(dx^2+e^{2x}dy^2+e^{2x}dz^2)+\ (ct+d)^{\frac{8n}{(n+3)(q+1)}}dm^2 \tag{37}$$

Now by eq. (33) and (36) in eq. (29), the scalar field can be written as

$$\phi=\phi_0\left(\frac{q+1}{q-3}\right)(ct+d)^{\frac{q-3}{q+1}}+k, \tag{38}$$

Where $\phi_0$and k are constants of integration.

The obtained directional Hubble parameters are given by,

$$H_1 = H_2 = \frac{4c}{(n+3)(q+1)(ct+d)} \tag{39}$$

$$H_3 = \frac{4nc}{(n+3)(q+1)(ct+d)} \tag{40}$$

Average Hubble parameter,

$$H = \frac{c}{(q+1)(ct+d)} \tag{41}$$

Spatial Volume,

$$V = a^4 = (ct+d)^{\frac{1}{(q+1)}} \tag{42}$$

The expansion scalar,

$$\theta = \frac{4c}{(q+1)(ct+d)} \tag{43}$$

The shear scalar,

$$\sigma^2 = \frac{6(n-1)^2c^2}{(q+1)^2(ct+d)^2(n+3)^2} \tag{44}$$

The Anisotropy parameter,

$$A_h = \frac{3(n-1)^2}{(n+3)^2}$$

The ratio,

$$\frac{\sigma^2}{\theta^2} = \frac{3(n-1)^2}{8(n+3)^2}$$

Now, to find out the value of string density we will use this assumption (Vinutha et al. [47])

$$\lambda = \alpha\rho \tag{45}$$

$$p = \varkappa\rho \tag{46}$$

Where α, χ are examined as non-evolving state parameter. Integrating eq. (31) with the help of eq. (39),(41),(45) and (46), string density can be indicating as

$$\rho = \rho_0 \frac{1}{(ct+d)^{4[\frac{(\eta+1)(n+3)+\alpha}{(n+3)(q+1)}]}} \tag{47}$$

Where $\rho_0$is the rest energy density at the present era. So, we have get string tension density and pressure as

$$\lambda = \alpha\rho_0 \frac{1}{(ct+d)^{4[\frac{(\eta+1)(n+3)+\alpha}{(n+3)(q+1)}]}} \tag{48}$$

$$p = \chi\rho_0 \frac{1}{(ct+d)^{4[\frac{(\eta+1)(n+3)+\alpha}{(n+3)(q+1)}]}} \tag{49}$$

From eq. (27) using eq. (36), (38), (39), (40) and (47), we get the dark energy density as

$$\rho_{de} = \frac{48c^2(n+1)}{(n+3)^2(q+1)^2(ct+d)^2} - 3(ct+d)^{\frac{-8}{(n+3)(q+1)}} + \frac{1}{2}\omega\phi_0(ct+d)^{\frac{-8}{(q+1)}} - \rho_0 \frac{1}{(ct+d)^{4\left[\frac{(\eta+1)(n+3)+\alpha}{(n+3)(q+1)}\right]}} \quad (50)$$

Now, from the deviation free part of the eq. (32) we has been calculating the EoS parameter by using eq. (41) and (50),

$$\omega_{de} = -\frac{1}{\rho_{de}}\left[\frac{24c^2(n+1)(1-q)}{(n+3)^2(q+1)^2(ct+d)^2} - 3(ct+d)^{\frac{-8}{(n+3)(q+1)}}\frac{(n+1)}{(n+3)} - \frac{1}{2}\omega\phi_0(ct+d)^{\frac{-8}{(q+1)}} + \rho_0\frac{1}{(ct+d)^{4\left[\frac{(\eta+1)(n+3)+\alpha}{(n+3)(q+1)}\right]}}\left[-1+\frac{(\eta+1)(n+3)+\alpha}{(n+3)}\right]\right] \quad (51)$$

The value of skewness parameter β can be obtained by subtracting eq. (25) from (24) and we get

$$\beta = \frac{\lambda}{\rho_{de}} \quad (52)$$

Put the value of λin eq. (52)

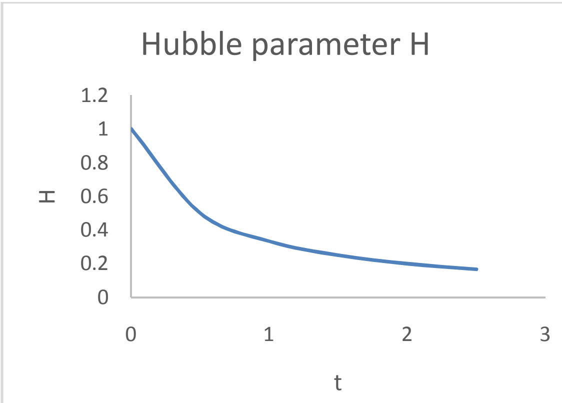
$$\beta = \frac{1}{\rho_{de}}\alpha\rho_0\frac{1}{(ct+d)^{4\left[\frac{(\eta+1)(n+3)+\alpha}{(n+3)(q+1)}\right]}} \quad (53)$$

Subtracting the eq. (26) from (25) and we get

$$\beta = \gamma \quad (54)$$

Put the value of β in eq. (54)

$$\gamma = \frac{1}{\rho_{de}}\alpha\rho_0\frac{1}{(ct+d)^{4\left[\frac{(\eta+1)(n+3)+\alpha}{(n+3)(q+1)}\right]}} \quad (55)$$



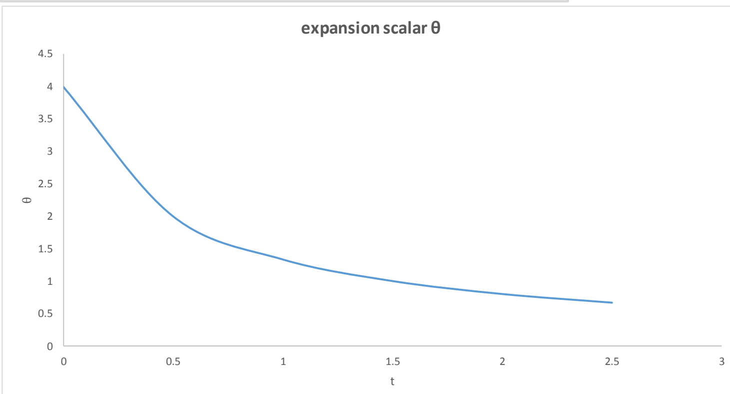


Figure:1 Graphical representation of Hubble Parameter and cosmic time

Figure:2 Graphical representation of expansion scalar verses cosmic time

V Discussion and conclusions

The anisotropic parameter is independent of time and $A_h \neq 0$ for $n \neq 1, A_h = 0\ for\ n = 1$. In this paper the model is anisotropic throughout. The spatial volume increases and becomes infinitely large as $t \to \infty$. We observe that $H, \sigma^2, \theta, \rho, p, \lambda$ diverge at t=0 and vanish at $t \to \infty$. The model presented here is anisotropic, shearing and accelerating. The early stage of evolution and the present universe. The skewness parameters have same values. The EoS parameter appear to be time dependent.

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Ganita Sandesh Vol. 37 (June & December, 2023) pp. 29 - 38 ISSN : 0970-9169

A METHOD FOR SOLVING FUZZY TRANSPORTATION PROBLEM

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ABSTRACT: The most important and successful applications in the optimization refers to transportation problem. The main aspect of this paper is to find the least transportation cost of some commodities through a capacitated network when the supply and demand of nodes and the capacity and cost of edges are represented as fuzzy numbers. Here, we are solving the transportation problem using the Robust ranking technique, where fuzzy demand and supply all are in the form of trapezoidal fuzzy numbers. The fuzzification of the cost of the transportation problem is discussed with the help of a numerical example.

KEY WORDS: Trapezoidal fuzzy numbers, Fuzzy transportation problem, Robust ranking technique.

1. Introduction

The transportation problem is one of the earliest applications of linear programming problems. Transportation models have wide applications in logistics and supply chain for reducing the cost efficient algorithms have been developed for solving the transportation problem when the cost coefficients and the supply and demand quantities are known exactly. The occurrence of randomness and imprecision in the real world is inevitable owing to some unexpected situations. There are cases that the cost coefficients and the supply and demandquantities of a transportation problem may be uncertain due to some uncontrollable factors To deal quantitatively with imprecise information in making decisions Bellman et.al.[2] introduced the notion of fuzziness.

Let a_i be the number of units of a product available at origin$i$ and b_j be the number of units of the product required at destination$j$. Let C_{ij} be the cost of transporting one unitfrom originito destination $j$and let X_{ij} be the amount of quantity carried or shipped fromorigin$i$ to destination j. There are effective algorithms for solving the transportation problems when all the decision parameters, i.e. the supply available at each source, the demand required at each destination as well as the unit transportation costs are given in a precise way. But in real life, there are many diverse situations due to uncertainty in one or more decision parameters and hence they may not be expressed in a precise way. This is due to measurement inaccuracy, lack of evidence, computational errors, high information cost, whether conditions etc. Hence we cannot apply the traditional classical methods to solve the transportation problems successfully. Therefore the use of Fuzzy transportation problems is more appropriate to model and solve the real world problems. A fuzzy transportation problem is a transportation problem in which the transportation costs, supply and demand are fuzzy quantities.

Bellman and Zadeh [3] proposed the concept of decision making in Fuzzy environment. After this pioneering work, several authors such as Shiang-Tai Liu and Chiang Kao[16], Chanas et.al[5], Pandianet.al [14], Liu and Kao [11] etc. proposed different methods for the solution of Fuzzy transportation problems. Chanas and Kuchta [4] proposed the concept of the optimal solution for the Transportation with Fuzzy coefficient expressed as Fuzzy numbers. Chanas, Kolodziejckzy, Machaj[5] presented a Fuzzy linear programming model for solving Transportation problem. Liu and Kao [11] described a method to solve a Fuzzy Transportation problem based on extension principle. Lin introduced a genetic algorithm to solve Transportation with Fuzzy objective functions. Srinivasan [18]-[23] described the new methods to solve fuzzy transportation problem.

NagoorGani and Abdul Razak [13] obtained a fuzzy solution for a two stage cost minimizing fuzzy transportation problem in which supplies and demands are trapezoidal fuzzy numbers. A.NagoorGani, Edward Samuel and Anuradha [7] used Arshamkhan's Algorithm to solve a Fuzzy Transportation problem. Pandian and Natarajan [14] proposed a Fuzzy zero point method for finding a Fuzzy optimal solution for Fuzzy transportationproblem where all parameters are trapezoidal fuzzy numbers.

2. PRELIMINARIES

In this section we define some basic definitions which will be used in this paper.

2.1 Definition-1

The characteristic function μ_A (x) of a crisp set $A \subseteq X$ assigns a value either 0 or 1 to each member in X. This function can be generalized to a function $\mu_{\tilde{A}}(x)$ such that the value assigned to the element of the universal set X fall within a specified range ie.$\mu_{\tilde{A}} X \rightarrow [0,1]$. The assigned value indicate the membership grade of the element in the set A. The function$\mu_{\tilde{A}}(x)$ is called the membership function and the set

$\tilde{A}$ = {(x, μ_A(x)):$x \in A$ and μ_A(x)$\in$ $[0,1]$} is called a fuzzy set.

2.2 Definition-2

A fuzzy set A, defined on the set of real numbers R is said to be a fuzzy number if its membership function μ_A : R→ [0,1]has the following characteristics

(i) A is normal. It means that there exists an x ∈ R such that $\mu_A(x) = 1$

(ii) A is convex. It means that for every $x_1,x_2 \in R\mu_A(\lambda x_1+(1-\lambda)x_2) \geq \min\{\mu_A(x_1),\mu_A(x_2)\}$, $\lambda \in [0,1]$

(iii) $\mu_A$is upper semi-continuous.

(iv) Supp (A) is bounded in R.

2.3 Definition-3

A fuzzy number A is said to be non-negative fuzzy number if and only $\mu_A(x) = 0,\ \forall x < 0$

2.4 Definition-4

A fuzzy number $\tilde{A}$ = (a, b, c, d) said to be a trapezoidal fuzzy number if its membership function is given by, where $a \leq b \leq c \leq d$.

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, & x < a, \\ \frac{x-a}{b-a}, & a < x \leq b, \\ 1, & b < x < c, \\ \frac{d-x}{d-c}, & c \leq x < d, \\ 0, & x > d \end{cases}$$

2.5 Definition-5

A trapezoidal fuzzy number $\tilde{A}$ = (a,b,c,d) is said to be non-negative (non-positive) trapezoidal fuzzy number i.e. A≥0(A≤0) if and only if a≥0(c≤0). A trapezoidal fuzzy number is said to be positive (negative) trapezoidal fuzzy number i.e. A > 0(A<0) if and only if a>0 (c>0).

2.6 Definition-6

Two trapezoidal fuzzy number $\tilde{A}_1$ = (a,b,c,d) and $\tilde{A}_2$ = (e,f,g,h) are said to be equal i.e. $\tilde{A}_1 = \tilde{A}_2$ if and only if a=e,b=f,c=g,d=h

2.7 Definition-7

Let$\tilde{A}_1$ = (a,b,c,d) and $\tilde{A}_2$ = (e,f,g,h) be two non-negative trapezoidal fuzzy number then

i. $\tilde{A}_1 \oplus \tilde{A}_2$ = (a,b,c,d) ⊕ (e,f,g,h) = (a+e,b+f,c+g,d+h)

ii. $\tilde{A}_1 - \tilde{A}_2$= (a,b,c,d) –(e,f,g,h) = (a-h,b-g,c-f,d-e)

iii. -$\tilde{A}_1$= -(a,b,c,d) = (-d,-c,-b,-a)

iv. $\tilde{A}_1 \otimes \tilde{A}_2$ = (a,b,c,d) ⊗(e,f,g,h) = (ae,bf,cg,dh)

v. $\frac{1}{A} \cong (\frac{1}{d}, \frac{1}{c}, \frac{1}{b}, \frac{1}{a})$

2.8 Robust Ranking Technique

Roubast ranking technique which satisfy compensation, linearityand additivity properties and provides results which are consist human intuition, If $\tilde{\alpha}$is a fuzzy number then the Roubast ranking is defined byR($\tilde{\alpha}$)= $\int_0^1 0.5(a_\alpha^L \, a_\alpha^U)d\alpha$,Where $(a_\alpha^l a_\alpha^U)$ is the α level cut of the fuzzy number $\tilde{\alpha}$and $(\alpha_\alpha^l \alpha_\alpha^U)$ = {((b-a)+a), (d-(d-c))}

In this paper we use this method for ranking the objective values. The Roubast ranking index R($\tilde{\alpha}$) gives the representative value of fuzzy number $\tilde{\alpha}$.

3. Mathematical formulation of a fuzzy transportation problem

Mathematically a transportation problem can be stated as follows:

Minimize

Z = $\sum_{i=1}^{m} \sum_{j=1}^{m} c_{ij} \, x_{ij}$ ……………….(1)

Subject to

$$\left.\begin{aligned} \sum_{j=1}^{n} x_{ij} &= \mathrm{a_i} \;\; \mathrm{j} = 1,2,\dots\dots \mathrm{n} \\ \sum_{i=1}^{m} x_{ij} &= \mathrm{b_j} \;\; \mathrm{i} = 1,2,\dots\dots \mathrm{m} \\ x_{ij} \;\geq 0 \;\; \mathrm{i} = 1,2,\dots\dots \mathrm{m}, \mathrm{j} &= 1,2,\dots\dots \mathrm{n} \end{aligned}\right\} \dots\dots(2)$$

Where C_{ij} is the cost of transportation of an unit from the i^{th} source to the j^{th} destination, and the quantity x_{ij} is to be some positive integer or zero , which is to be transported from the i^{th}origin to j^{th} destination. A obvious necessary and sufficient condition for the linear programming problem given in (1) to have a solution is that

$\sum_{i=1}^{n} a_i$= $\sum_{j=1}^{m} b_j$……………………..(3)

(i.e) assume that total available is equal to the total required. If it is not true, a fictitious source or destination can be added. It should be noted that the problem has feasible solution if and only if the condition (2) satisfied. Now, the problem is to determine x_{ij}, in such a way that the total transportation cost is minimum

Mathematically a fuzzy transportation problem can be stated as follows:

Minimize

$Z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij}\, x_{ij}$ …………..(4)

Subject to be

$$\left.\begin{aligned} \sum_{j=1}^{n} x_{ij} &= \tilde{a}_i \quad j = 1,2,\ldots\ldots n \\ \sum_{i=1}^{m} x_{ij} &= b_i \quad i = 1,2,\ldots\ldots m \\ x_{ij} &\geq 0 \quad i = 1,2,\ldots\ldots m,\ j = 1,2,\ldots\ldots n \end{aligned}\right\} \ldots\ldots.(5)$$

In which the transportation costs $\tilde{c}_{ij}$, supply a_i and demand $\tilde{b}_j$ quantities are fuzzy quantities . An obvious necessary and sufficient condition for the fuzzy linear programming problem give in (4-5) to have a solution is that

$\sum_{i=1}^{n} a_i = \sum_{j=1}^{m} \tilde{b}_j$………………(6)

This problem can also be represented as follows-

	1	……..	n	Supply
1	$\tilde{C}_{11}$	……..	$\tilde{C}_{1n}$	a_1
. . . .	. . . .	. . . .	. . . .	. . . .
M	$\tilde{C}_{m1}$	……..	$\tilde{C}_{mn}$	a_m
Demand	$\tilde{b}_1$	……..	$\tilde{b}_n$	

4. Alternate Method for Saving Transportation Problem

Following are the stags for solving Transportation Problem

Step-1: From the gives Transportation problem, convert fuzzy values to crisp values using ranking function.

Step-2 Deduct the minimum cell cost from each of the cell cost of every row/column of the Transportation problem and place them on the right-top/right-bottom of corresponding cost.

Step-3 Adding the cost of right-top and right - bottom and place the summation value in the corresponding cell cost.

Step-4 Identify the minimum element in each row and column of the Transportation table and subtract in their corresponding the row and the column.

Step-5: Find the sum of the values in the row and the column. Choose the maximum value and allocate the minimum of supply/demand in the minimum element of the row and column. Eliminate by deleting the columns or rows corresponding to where the supply or demand is satisfied.

Step-6: Continue step-4 and step -5 until satisfaction of all the supply and demand is met

Step-7 : Place the original transportation cost to satisfied cell cost.

Step-8: Calculate the minimum cost.

That is,

Total Cost = $\sum \sum C_{ij}X_{ij}$

5. Numerical Example

Consider the Fuzzy Transportation Problem

	FD_1	**FD_2**	**FD_3**	**FD_4**	**Fuzzy Capacity**
FO_1	[1,2,3,4]	[1,3,4,6]	[9,11,12,14]	[5,7,8,11]	[1,6,7,12]
FO_2	[0,1,2,4]	[-1,0,1,2]	[5,6,7,8]	[0,1,2,3]	[0,1,2,3]
FO_3	[3,5,6,8]	[5,8,9,12]	[12,15,16,19]	[7,9,10,12]	[5,10,12,17]
Fuzzy Demand	[5,7,8,10]	[1,5,6,10]	[1,3,4,6]	[1,2,3,4]	

Solution

In Conformation to model the fuzzy transportation problem can be formulated in the following mathematical programming form

Min Z = R(1, 2, 3, 4)x_{11} + R(1, 3, 4, 6)x_{12} + R(9, 11, 12, 14)x_{13} + R(5,7,8,11)x_{14} + R(0,1,2,4)x_{21} + R(-1,0,1,2)x_{22}+R(5,6,7,8)x_{23} +R(4,5,6,7)x_{24} + R(3,5,6,8)x_{31} + R(5,8,9,12)x_{32} + R(12,15,16,19)x_{33} + R(7,9,10,12)x_{34}

R($\tilde{a}$) = $\int_0^1 0.5(a_\alpha^L a_\alpha^U)d\alpha$

Where (a_α^L, a_α^U) = { ((b-a)+a),(d-(d-c))}

After applying ranking technique, we get

Table-1

	D1	D2	D3	D4	Supply
O1	2.5	3.5	11.5	7.75	6.5
O2	1.75	0.5	6.5	1.5	1.5
O3	5.5	8.5	15.5	9.5	11
Demand	7.5	5.5	3.5	2.5	

Table-2

	D1	D2	D3	D4	Supply
O1	2.5_0^0	3.5_3^1	11.5_5^9	$7.75_{6.25}^{5.25}$	6.5
O2	$1.75_{0.75}^{1.25}$	$.5_0^0$	6.5_0^6	1.5_0^1	1.5
O3	5.5_3^0	8.5_8^3	15.5_9^{10}	9.5_8^4	11
Demand	7.5	5.5	3.5	2.5	

Table-3

	D1	D2	D3	D4	Supply
O1	0	4	14	11.5	6.5
O2	2	0	6	1	1.5
O3	3	11	19	12	11
Demand	7.5	5.5	3.5	2.5	

With the help of the method, we get **Table-4**

	D1	D2	D3	D4	Supply
O1	0	5.5 4	1 14	11.5	6.5
O2	2	0	1.5 6	1	1.5
O3	7.5 3	11	1 19	2.5 12	11
Demand	7.5	5.5	3.5	2.5	

Finally, we get **Table-5**

	D1	D2	D3	D4	Supply
O1		5.5 3.5	1 11.5		6.5
O2			1.5 6.5	1	1.5
O3	7.5 5.5		1 15.5	2.5 9.5	11
Demand	7.5	5.5	3.5	2.5	

Hence (4+3-1)=6 cells are allocated and hence we got our feasible solution. Next we calculate total cost and its corresponding allocated value of supply and demand Total Cost (5.5X7.5)+(3.5X5.5)+(11.5X1)+(6.5X1.5)+(15.5X1) + (9.5X2.5) =121

This is a basic feasible solution. The solution obtained using NCM, LCM, VAM and MODI/Stepping stone methods respectively. Hence the basic feasible solution obtained from method is optional solution.

6. Conclusion

In this paper, the transportation costs are considered as imprecise numbers described by fuzzy numbers which are more realistic and general in nature. Moreover, the fuzzy transportation problem of trapezoidal fuzzy numbers has been transformed into crisp transportation problem using robust ranking technique indices. Numerical examples show that by this method we can have the optimal solution as well as the crisp and fuzzy optimal total cost. By using robust ranking method we have shown that the total cost obtained is optimal. Hence, this will be helpful for decision makers who are handling logistic and supply chain problems in fuzzy environment. For future research we propose effective implementation of the trapezoidal fuzzy numbers in all fuzzy problems.

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Vol. 37 (June & December) 2023

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